Reasoning with the Variable Parts and Multiple Batches Perspectives on Proportional Relationships

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Dean Stevenson, Torrey Kulow, and Ibrahim Burak Ölmez

University of Georgia

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Research Team

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- Merve Kursav
- Eric Siy

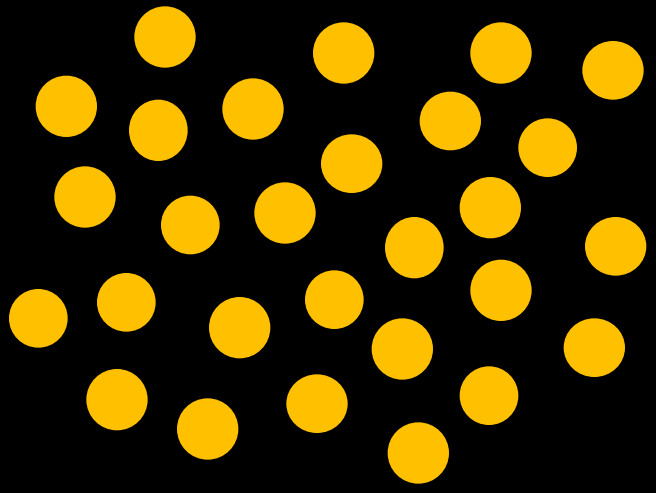
Other sessions:

- **Friday 10:15-11:30, Salon 7**: *The Development and Negotiation of Sociomathematical Norms for Drawings*

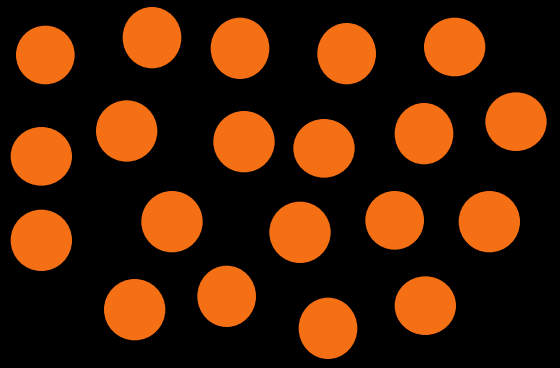
Please work on the following problem:

You make one piece of jewelry with 7 ounces of gold and 5 ounces of copper. You make a second piece of jewelry with 28 ounces of gold and 20 ounces of copper. Do the two pieces of jewelry have the same “quality”? Arrange the amounts of gold and copper to visually show whether or not the second piece has the same quality as the first.

28 ounces of gold



20 ounces of copper



TWO WAYS TO ORGANIZE A FIXED RATIO

MULTIPLE BATCHES

VARIABLE PARTS

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VARIABLE PARTS

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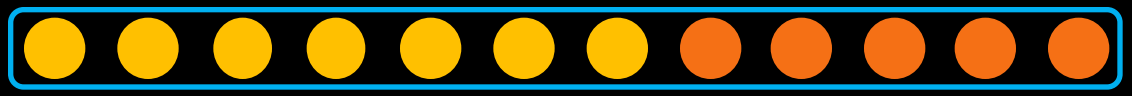

1 batch

MULTIPLE BATCHES

VARIABLE PARTS

TWO WAYS TO ORGANIZE A FIXED RATIO

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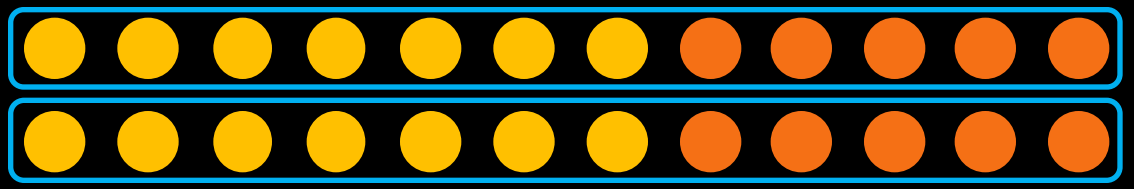

1 batch

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VARIABLE PARTS

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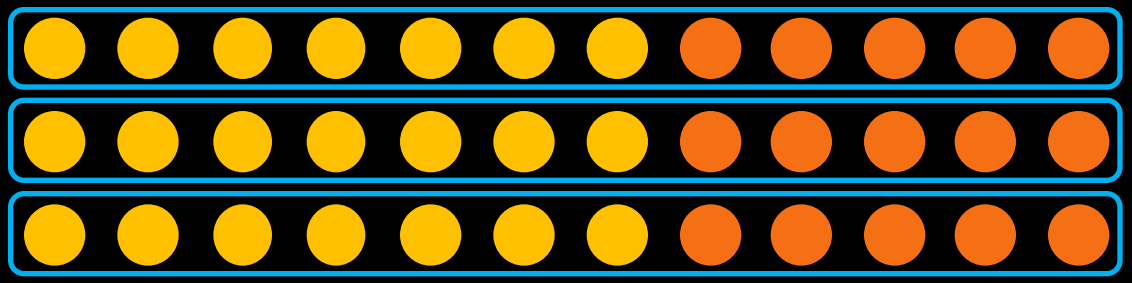

VARIABLE PARTS

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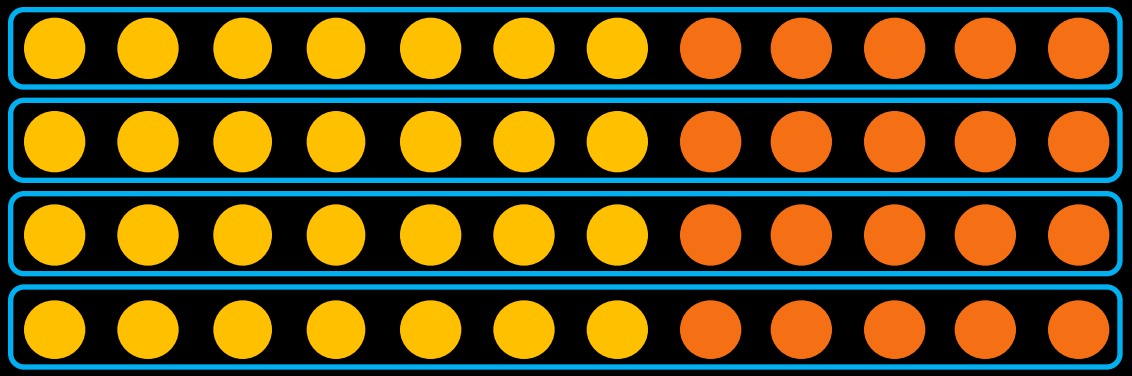

VARIABLE PARTS

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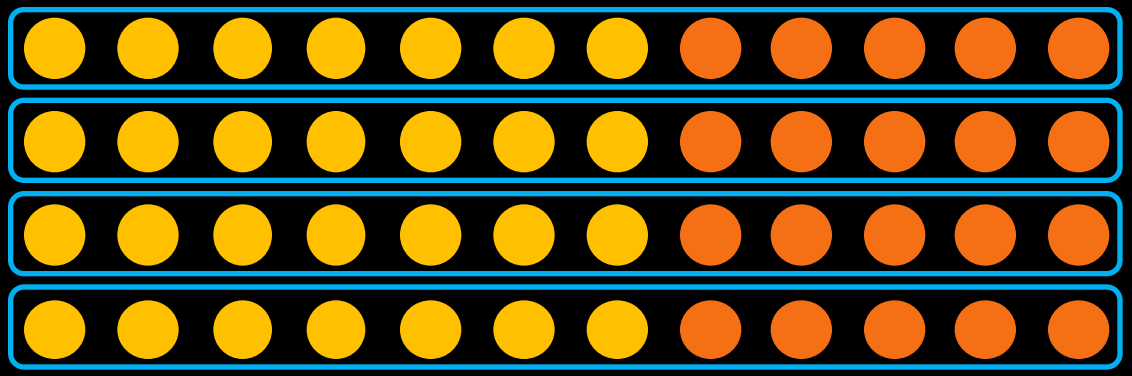

VARIABLE PARTS

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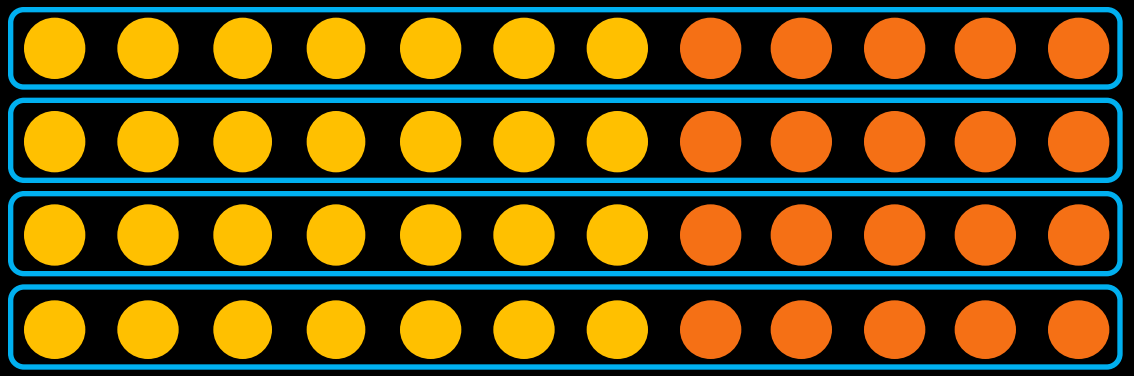
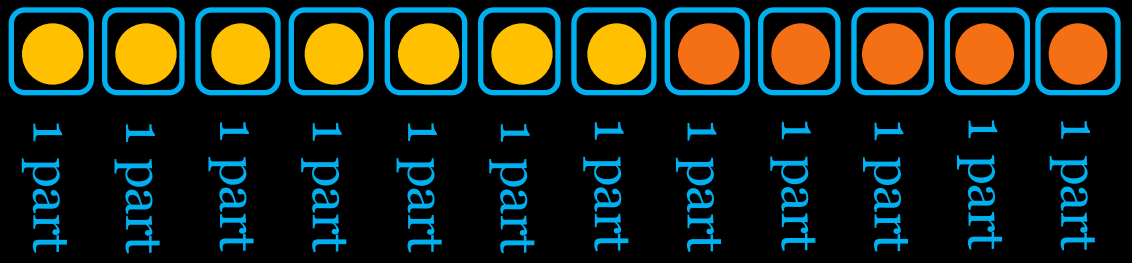


VARIABLE PARTS



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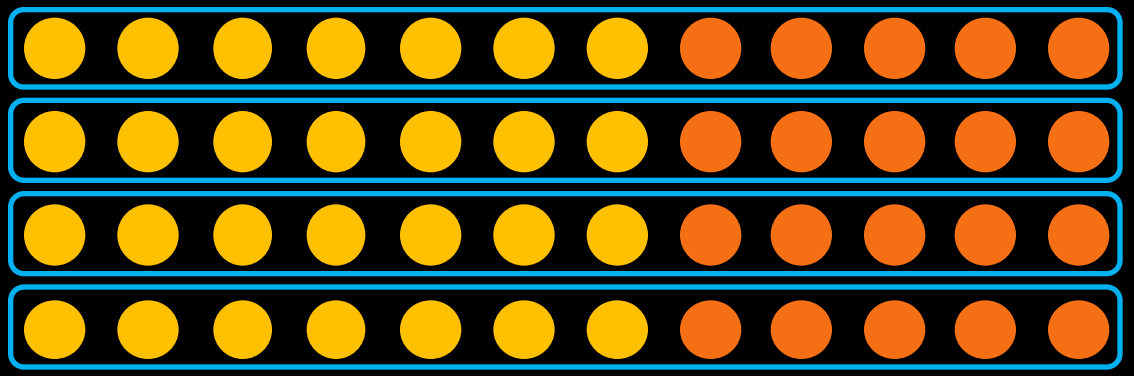

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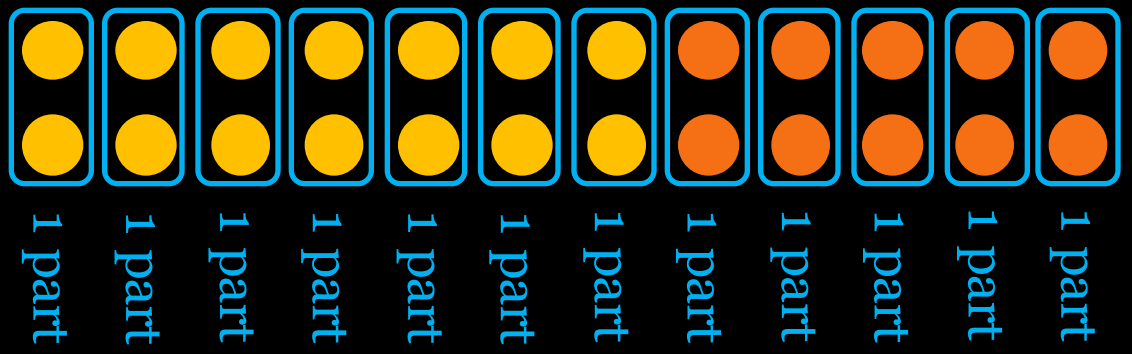

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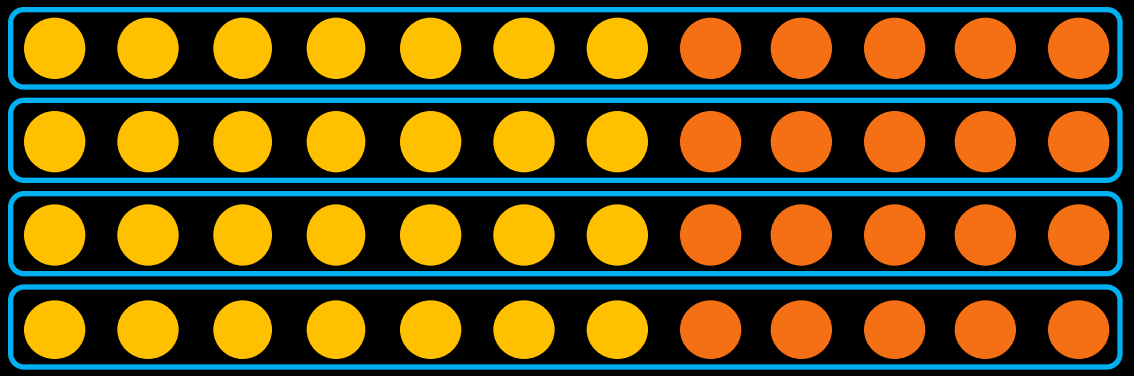

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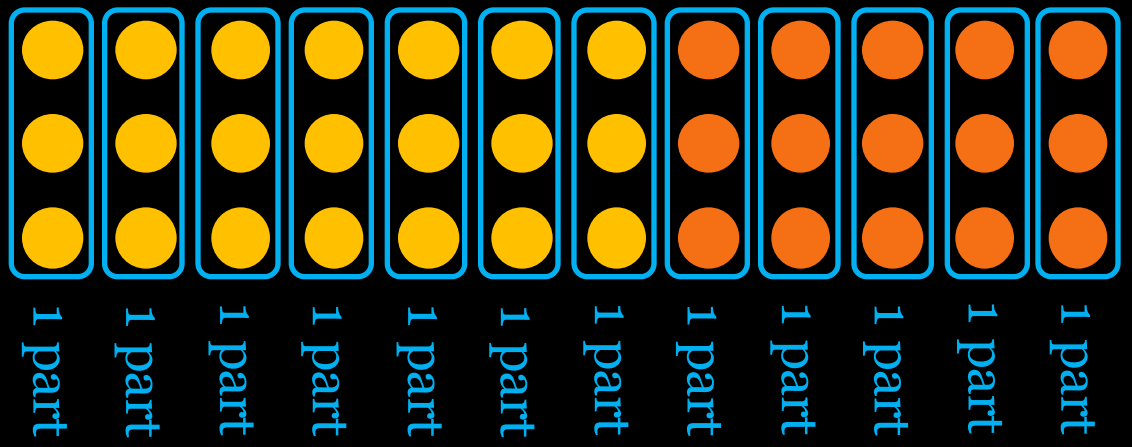

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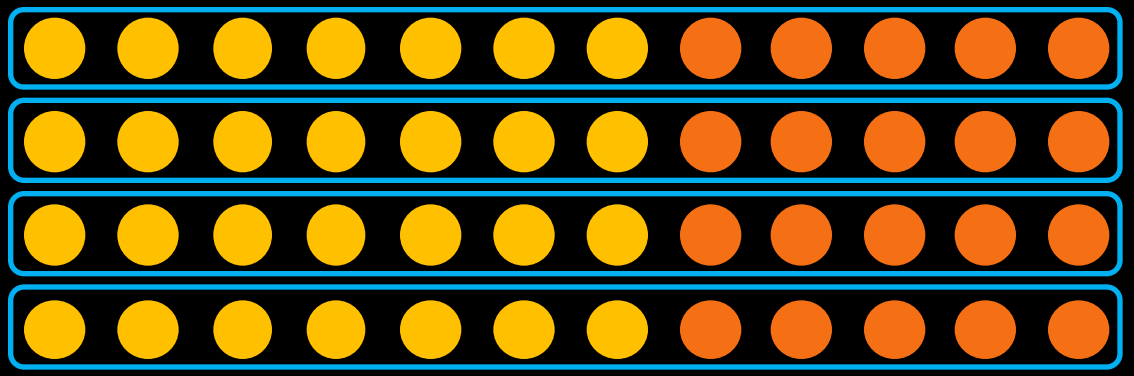

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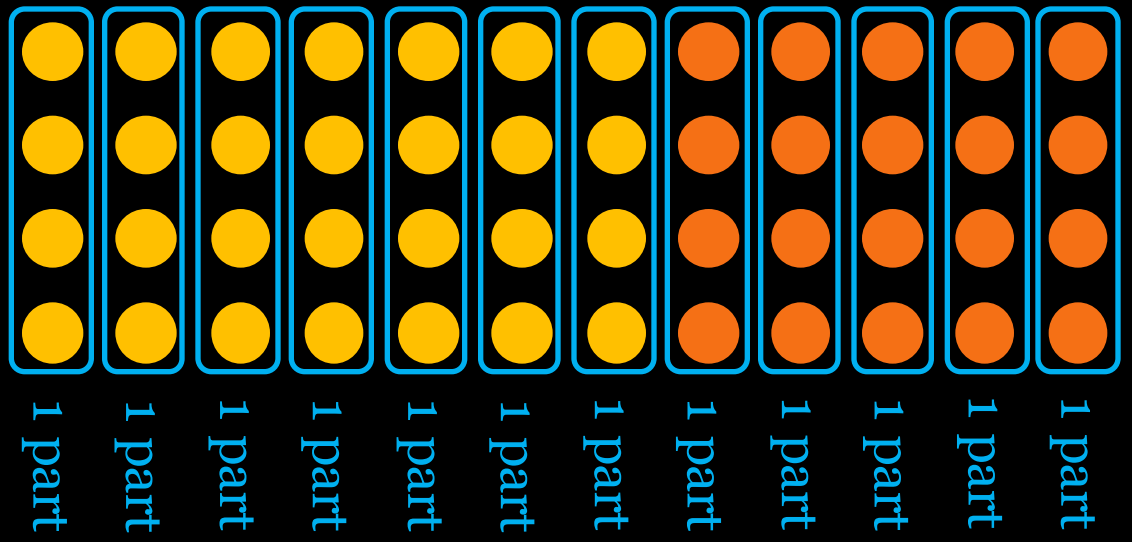


QUANTITATIVE MEANING OF MULTIPLICATION

MULTIPLIER
number of equal-size groups

MULTIPLICAND
number of base units in 1 group

PRODUCT
number of base units in M groups

$$M \bullet N = P$$

"how many groups" division

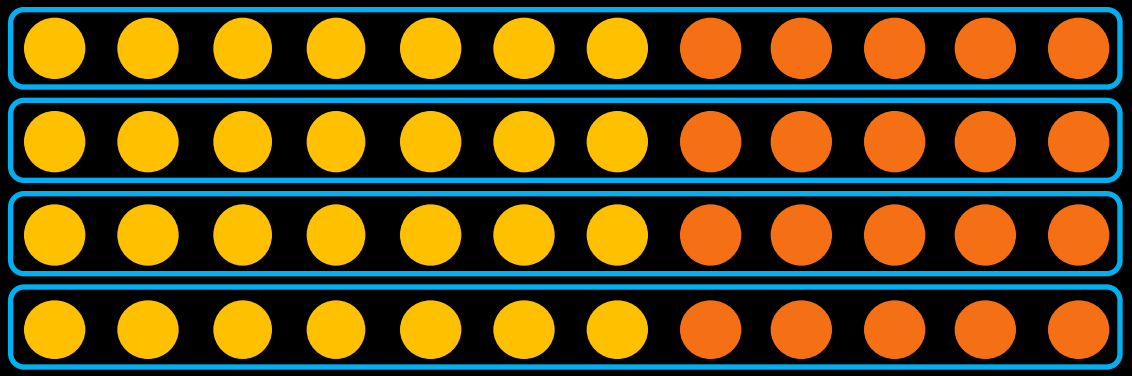
$$? \bullet N = P$$

"how many units in 1 group" division

$$M \bullet ? = P$$

GENERATE AN EQUATION FOR THE GOLD

MULTIPLE BATCHES



MULTIPLICAND
number of base units in 1 group

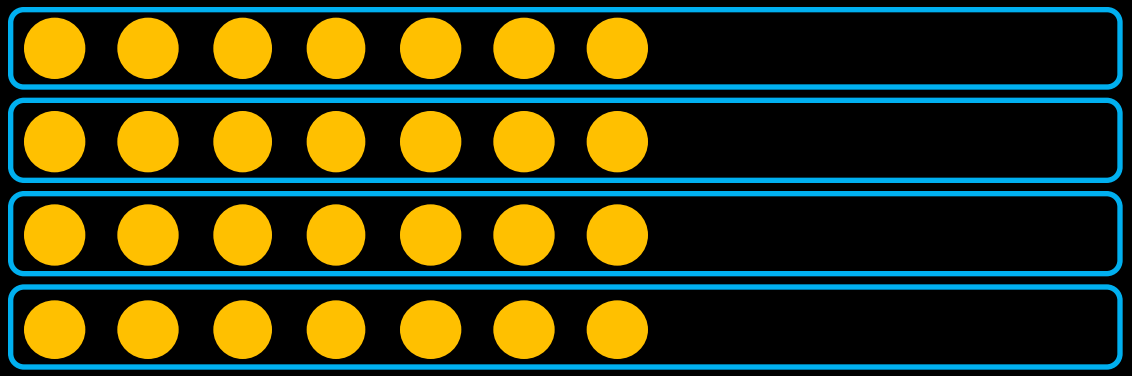
MULTIPLIER
number of equal-size groups

PRODUCT
number of base units in M groups

$$M \cdot N = P$$

GENERATE AN EQUATION FOR THE GOLD

MULTIPLE BATCHES



MULTIPLICAND
number of base units in 1 group

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number of base units in M groups

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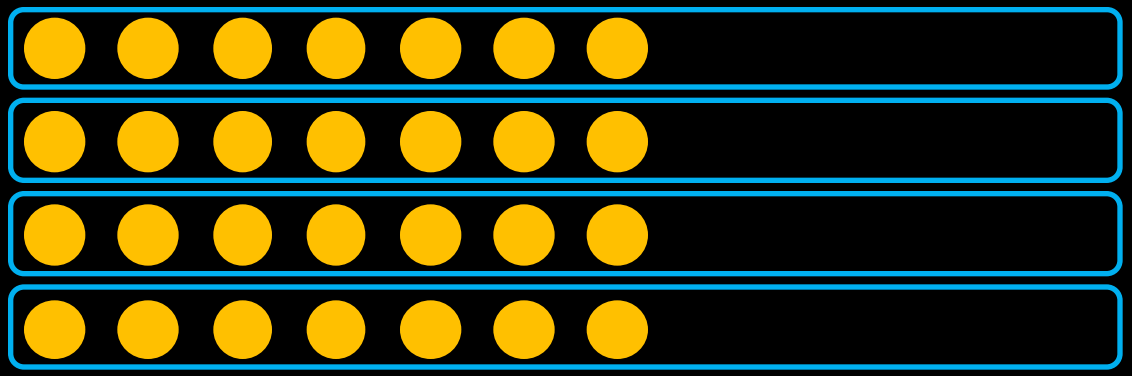
GENERATE AN EQUATION FOR THE GOLD

MULTIPLE BATCHES



MULTIPLICAND
number of gold ounces in 1 batch

MULTIPLIER
number of equal-size batches

PRODUCT
number of gold ounces in 7 batches

$$4 \cdot 7 = 28$$

GENERATE AN EQUATION FOR THE GOLD

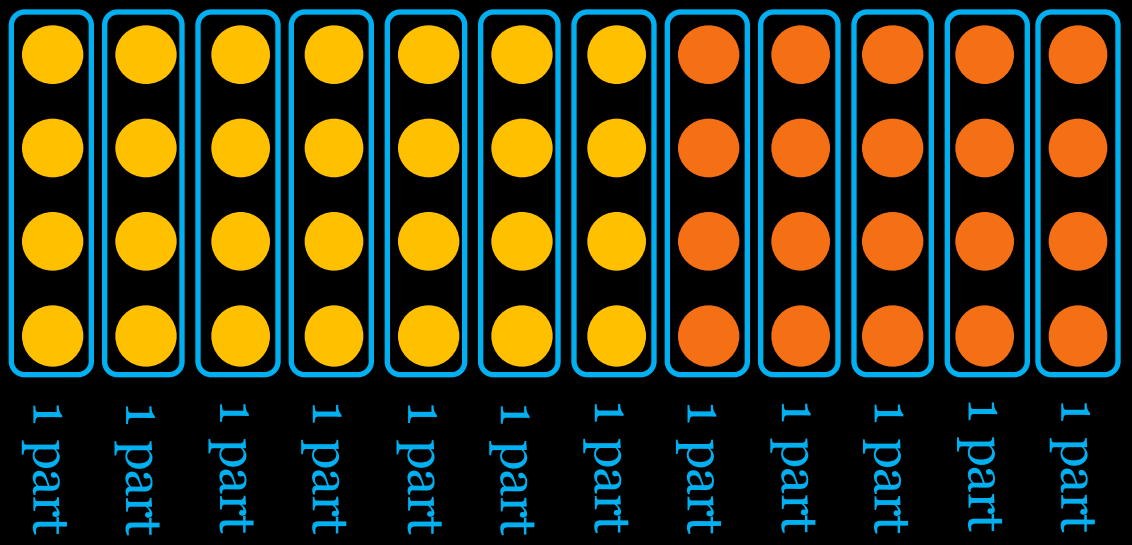
MULTIPLICAND
number of base units in 1 group

MULTIPLIER
number of equal-size groups

PRODUCT
number of base units in M groups

$$M \cdot N = P$$

VARIABLE PARTS



GENERATE AN EQUATION FOR THE GOLD

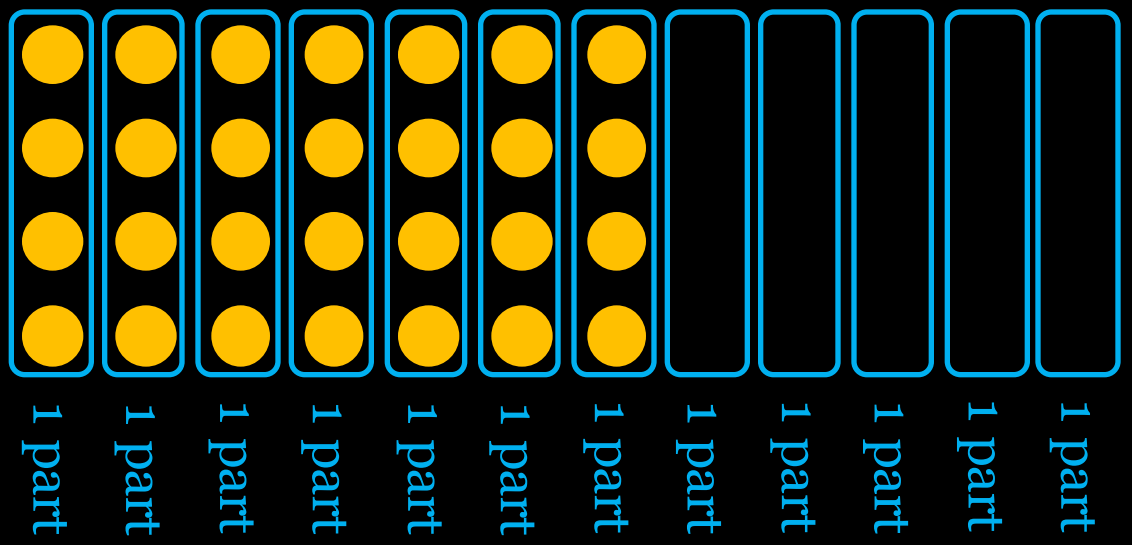
MULTIPLICAND
number of base units in 1 group

MULTIPLIER
number of equal-size groups

PRODUCT
number of base units in M groups

$$M \cdot N = P$$

VARIABLE PARTS



GENERATE AN EQUATION FOR THE GOLD

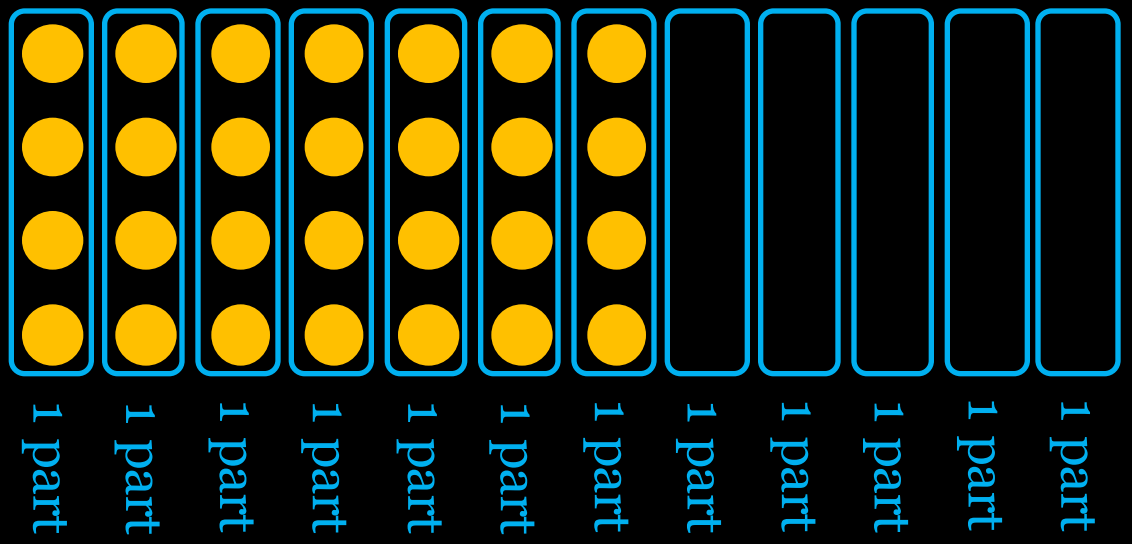
MULTIPLICAND
number of gold ounces in 1 part

MULTIPLIER
number of equal-size parts

PRODUCT
number of gold ounces in 7 parts

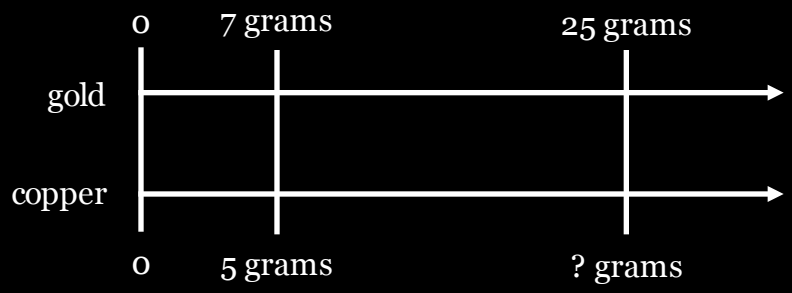
$$7 \cdot 4 = 28$$

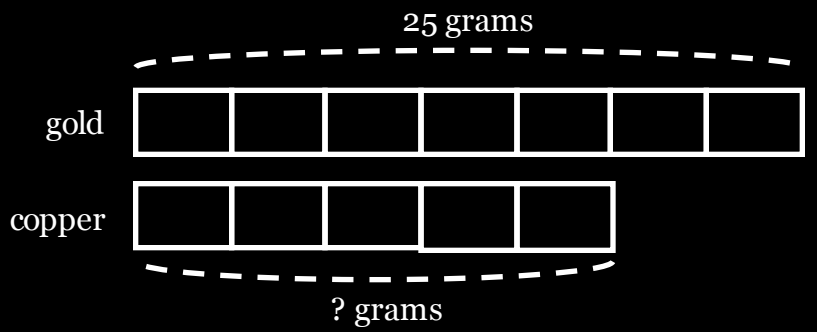
VARIABLE PARTS



Please work on the following problem:

A company makes jewelry gold using gold and copper. The company uses different weights of gold and copper on different days, but always in the same ratio of 7 to 5. If the company uses 25 grams of gold on one day, how much copper will they use?





TWO PERSPECTIVES ON PROPORTIONAL RELATIONSHIPS

MULTIPLE BATCHES STRATEGIES

VARIABLE PARTS STRATEGIES

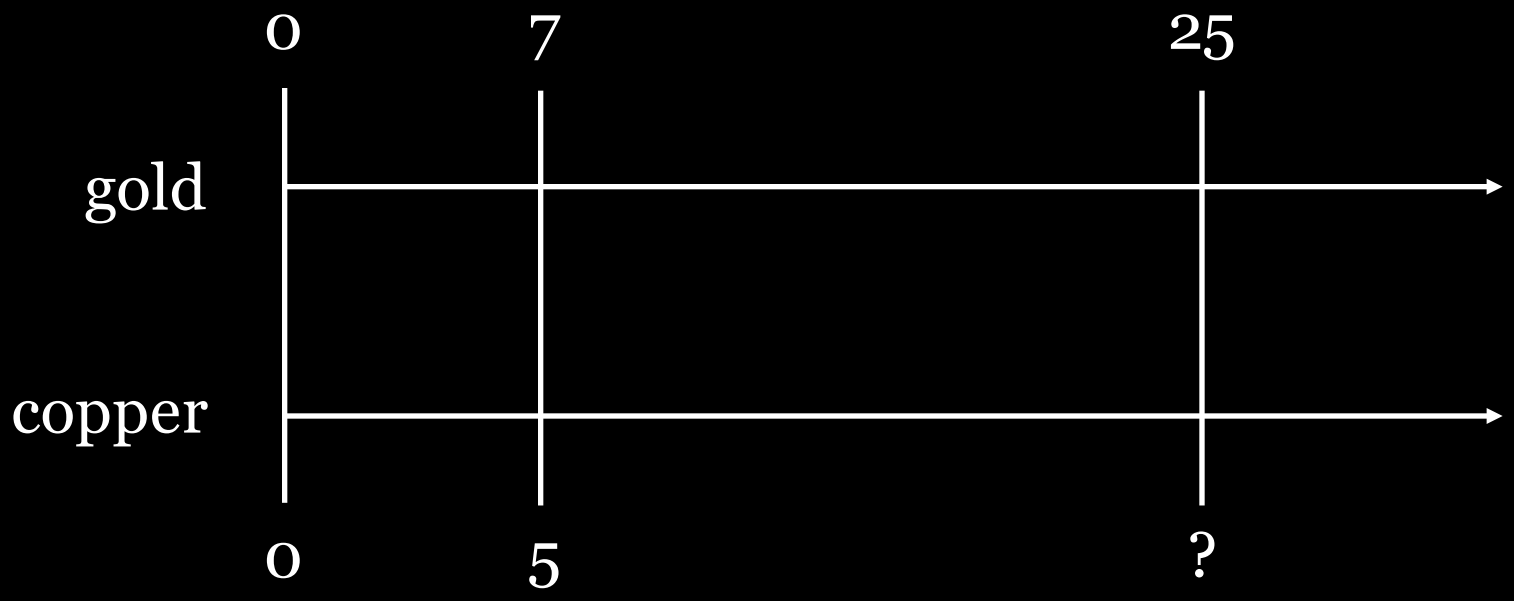
MULTIPLE BATCHES STRATEGIES

Multiply One Batch

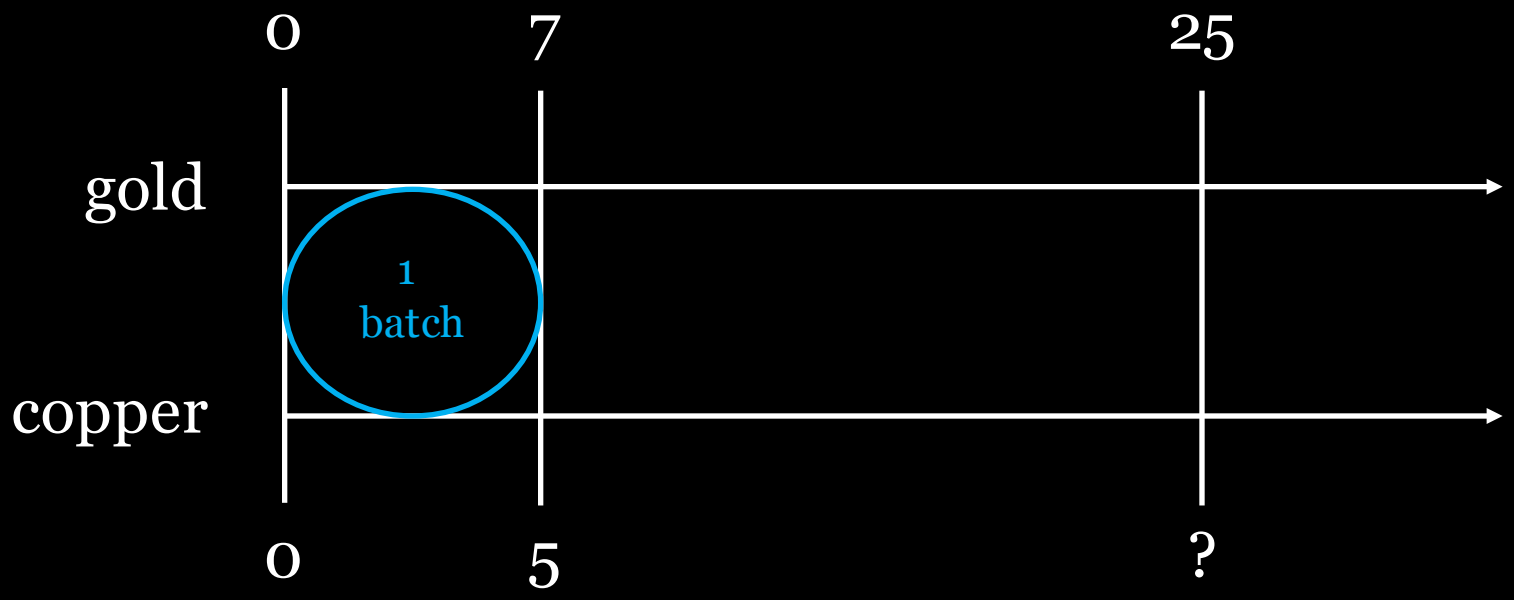
VARIABLE PARTS STRATEGIES

Multiply One Batch

Multiply One Batch



Multiply One Batch



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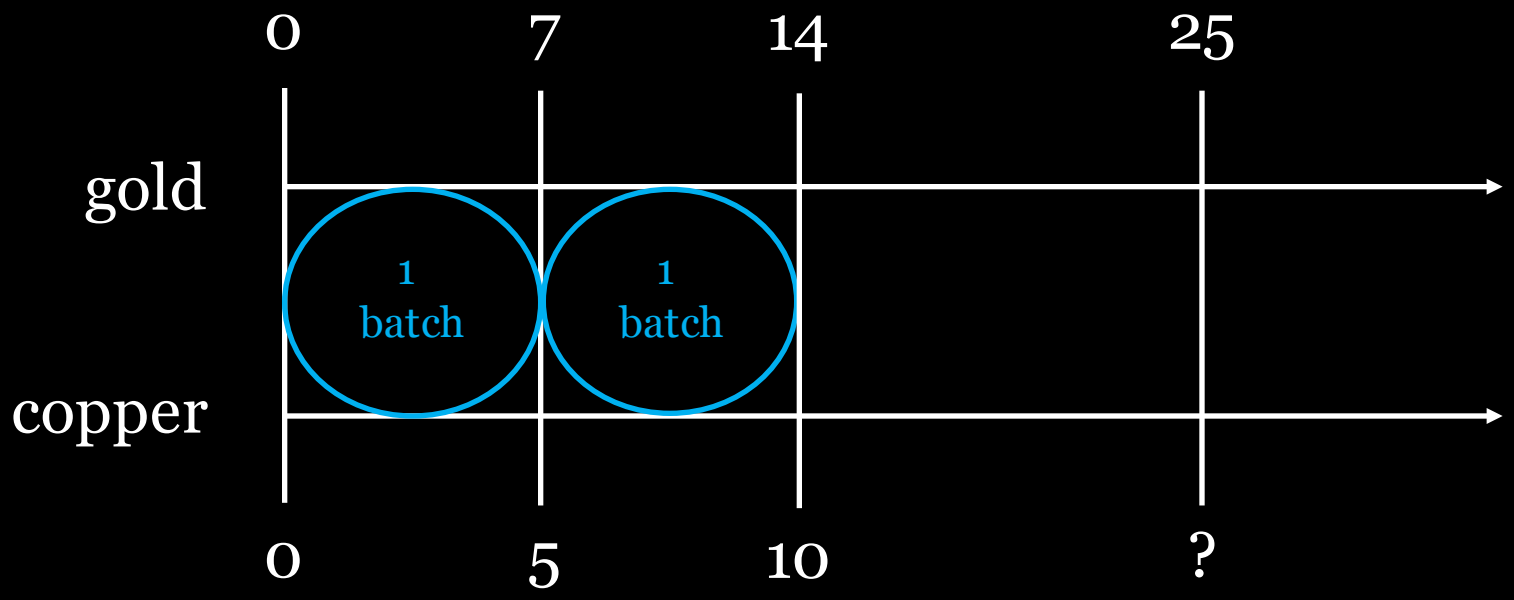

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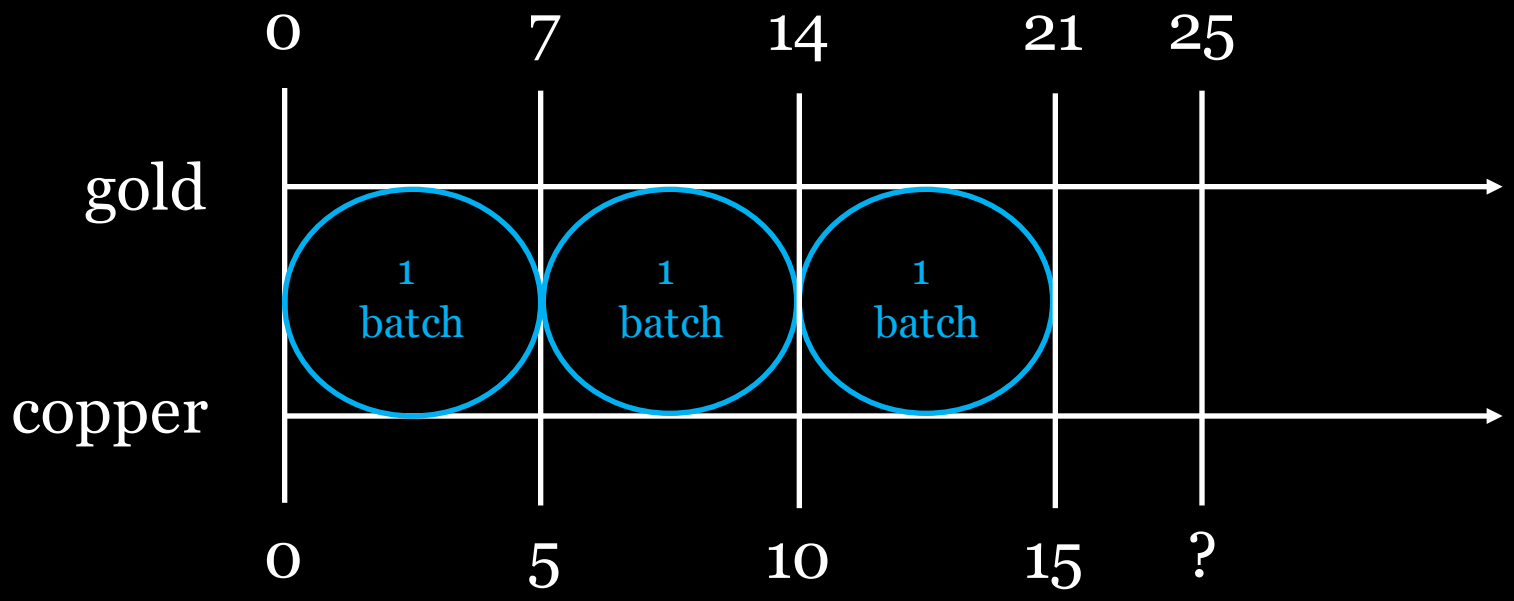

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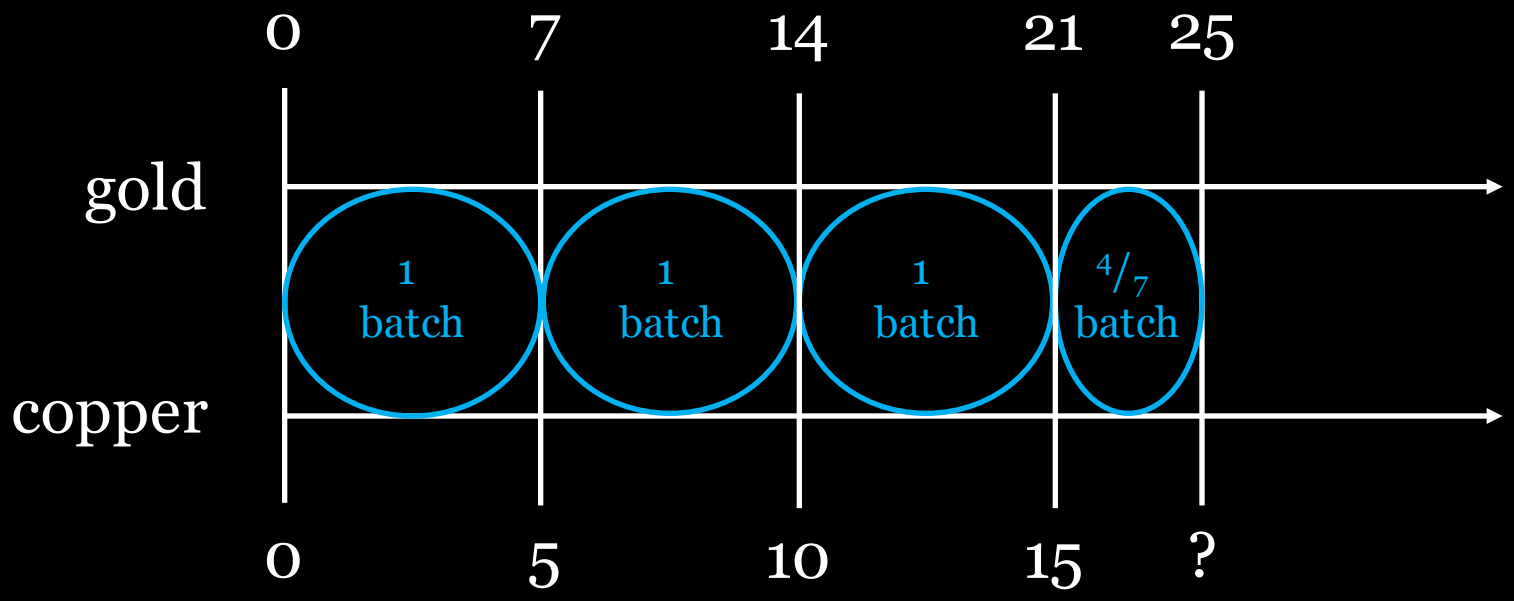
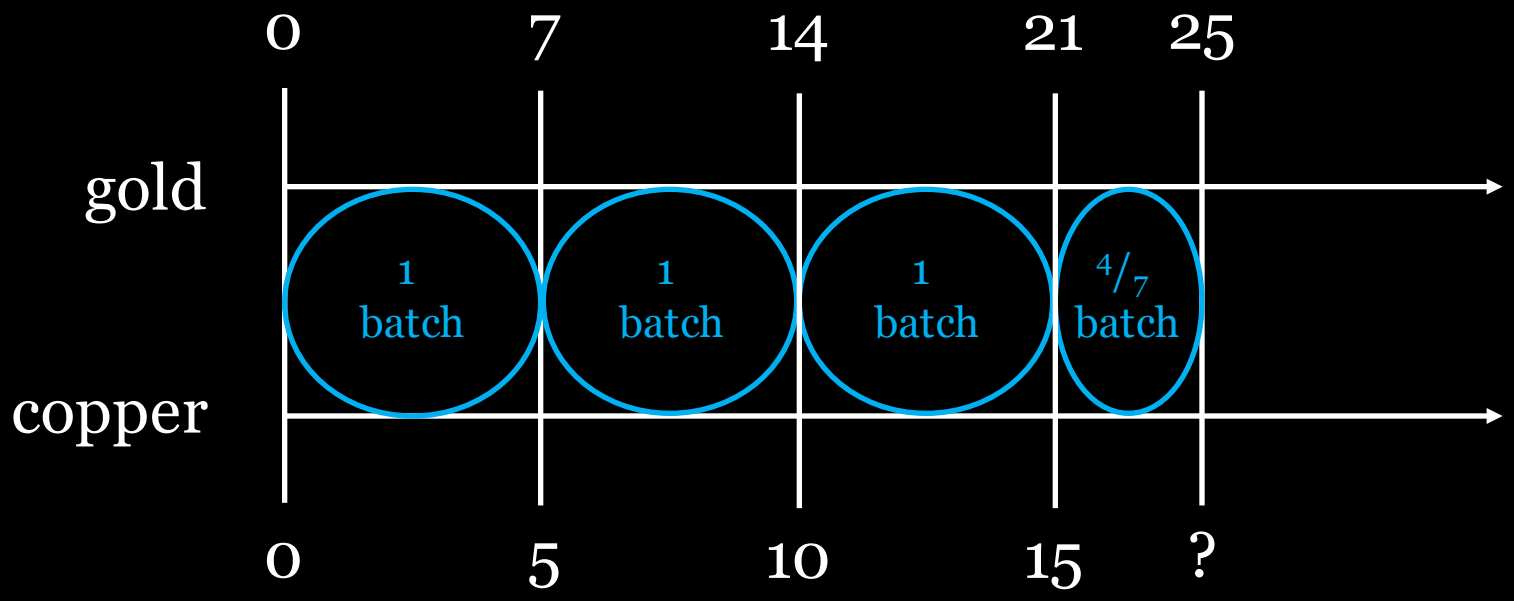
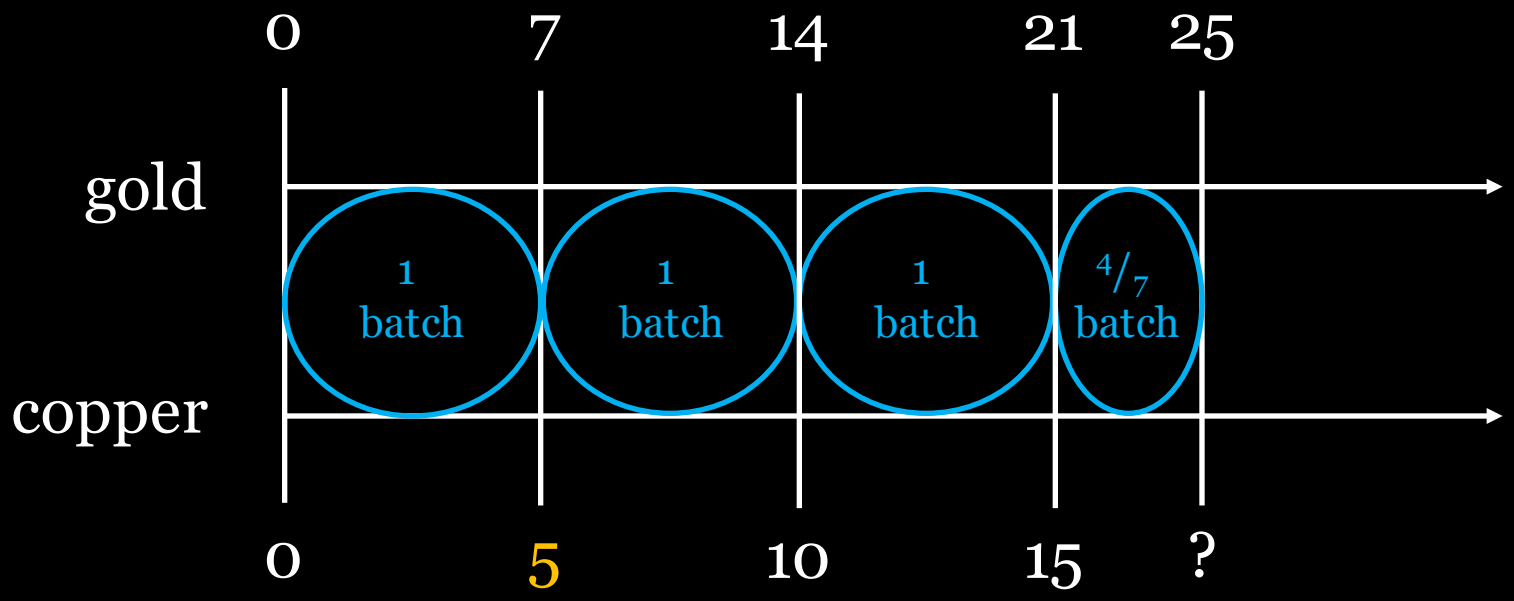


Multiply One Batch



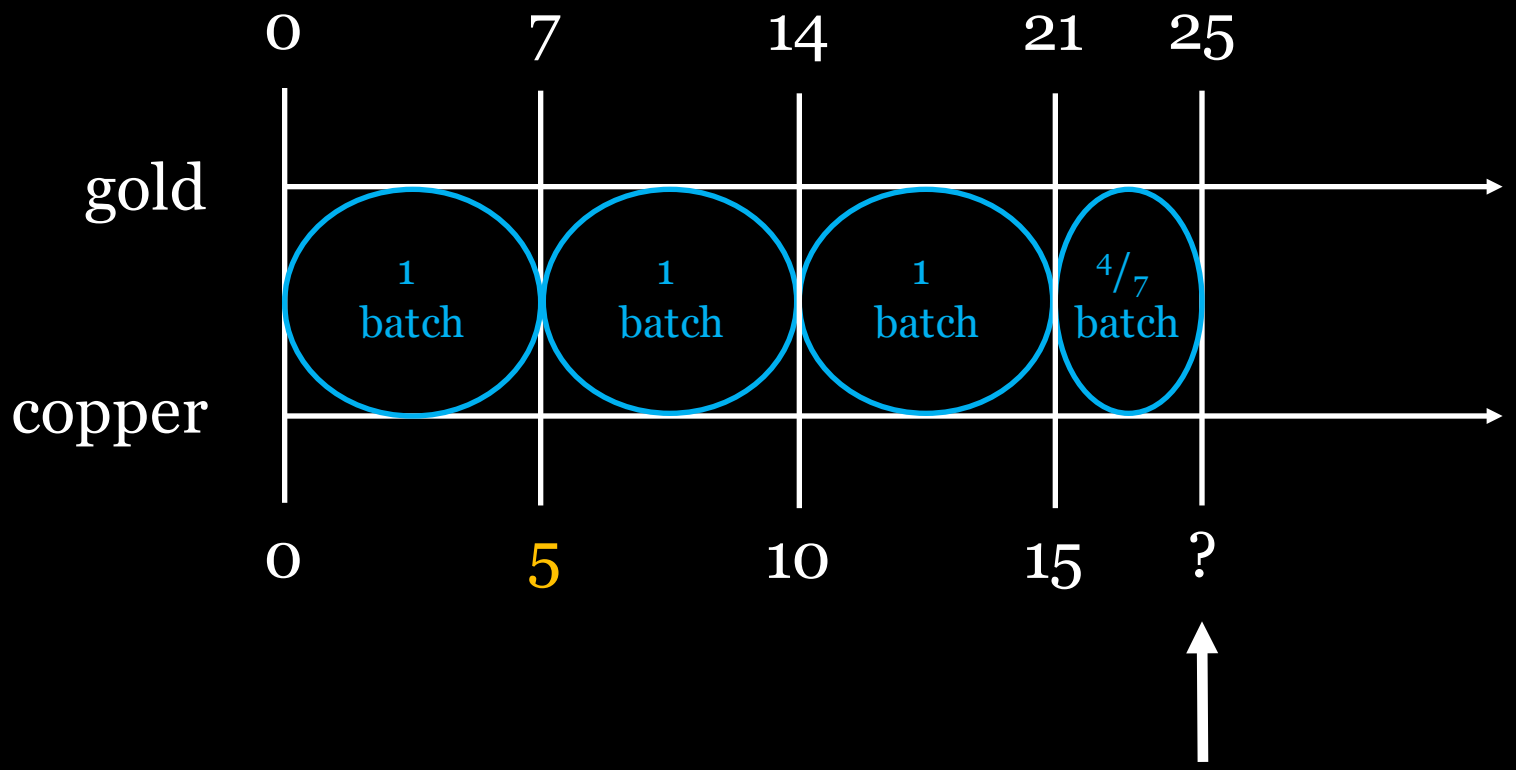
$^{25}/_{7}$ batches

Multiply One Batch



$^{25}/_{7}$ batches

Multiply One Batch



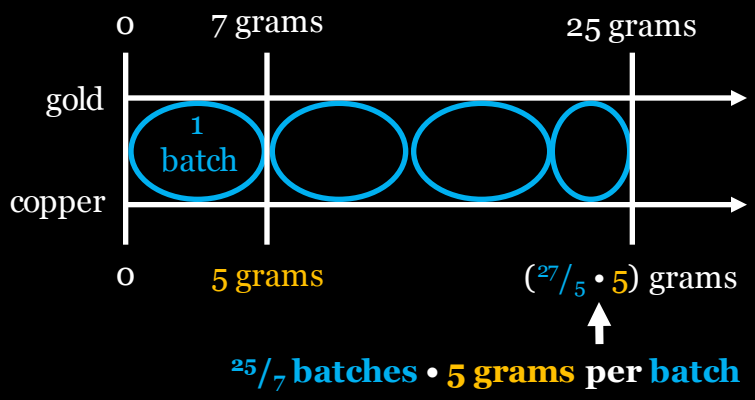
$^{25}/_{7}$ batches • 5 grams per batch

MULTIPLE BATCHES STRATEGIES

Multiply One Batch

VARIABLE PARTS STRATEGIES

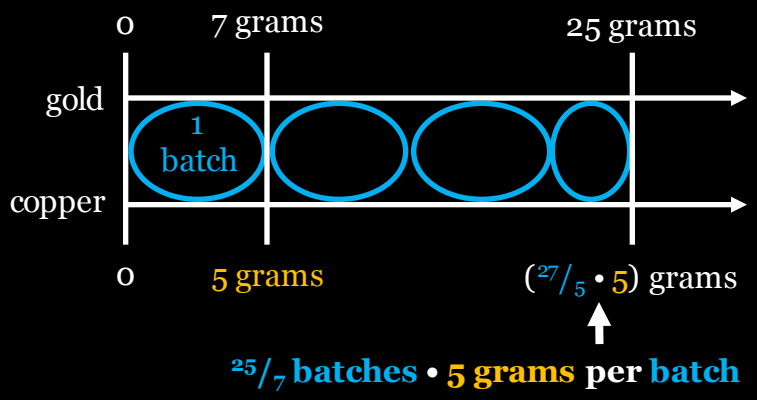
MULTIPLE BATCHES STRATEGIES



Multiply One Batch

VARIABLE PARTS STRATEGIES

MULTIPLE BATCHES STRATEGIES

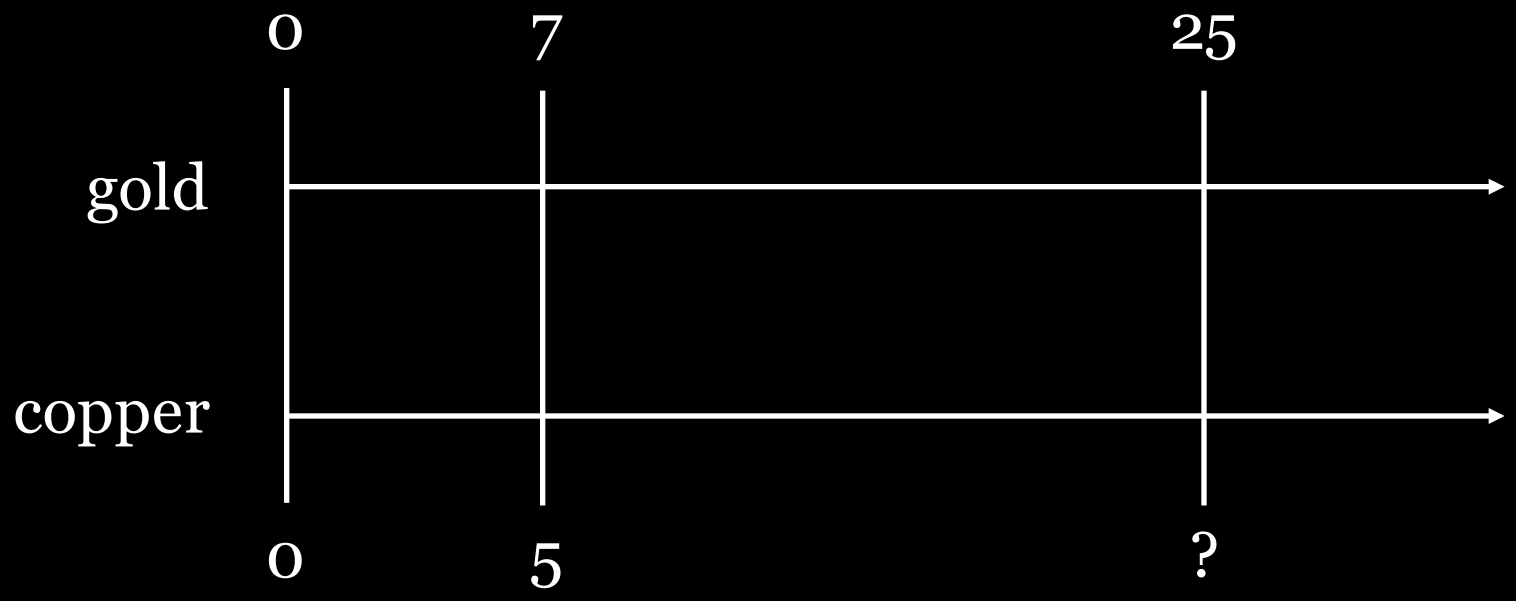


Multiply One Batch

Multiply Unit-Rate Batch

VARIABLE PARTS STRATEGIES

Multiply Unit-Rate Batch



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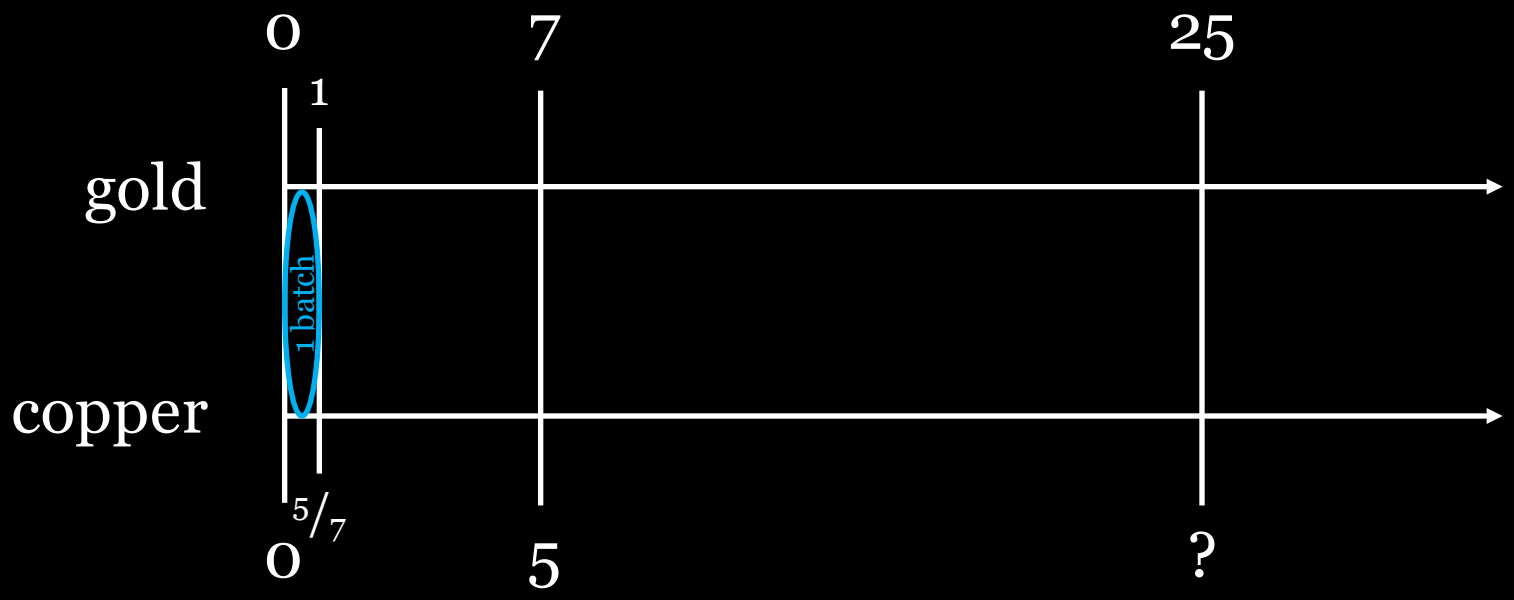

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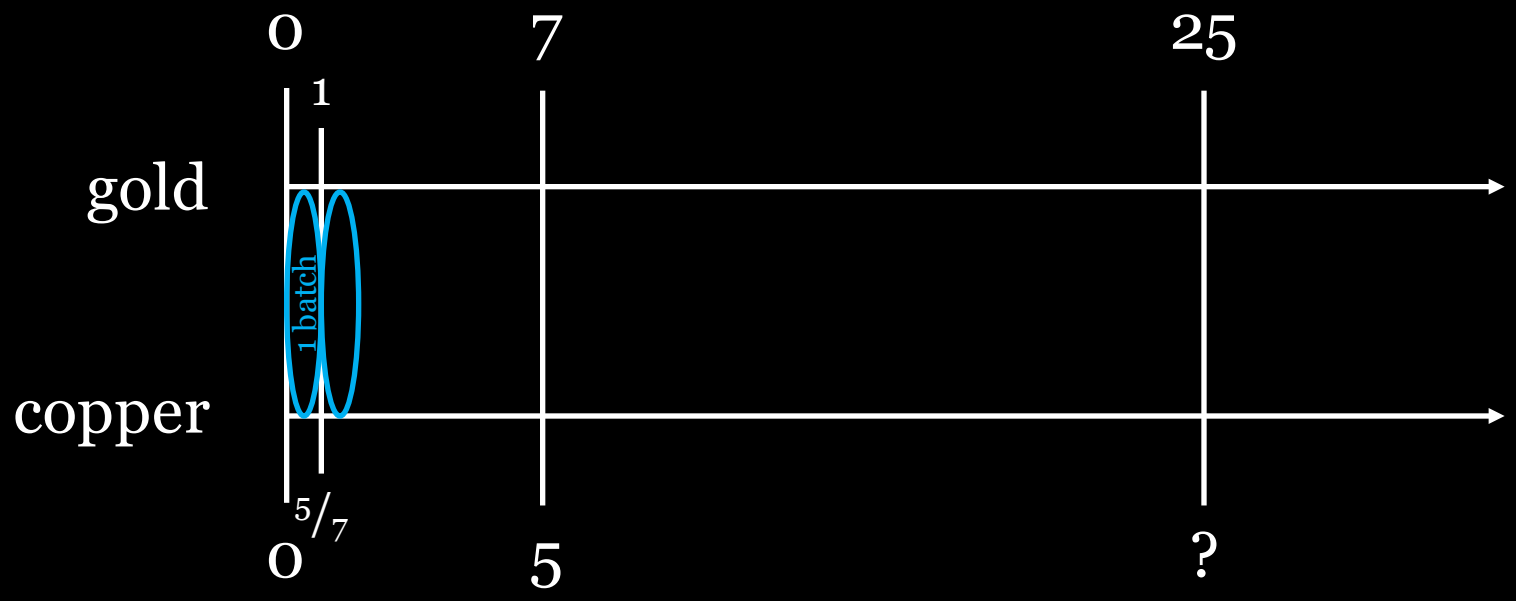

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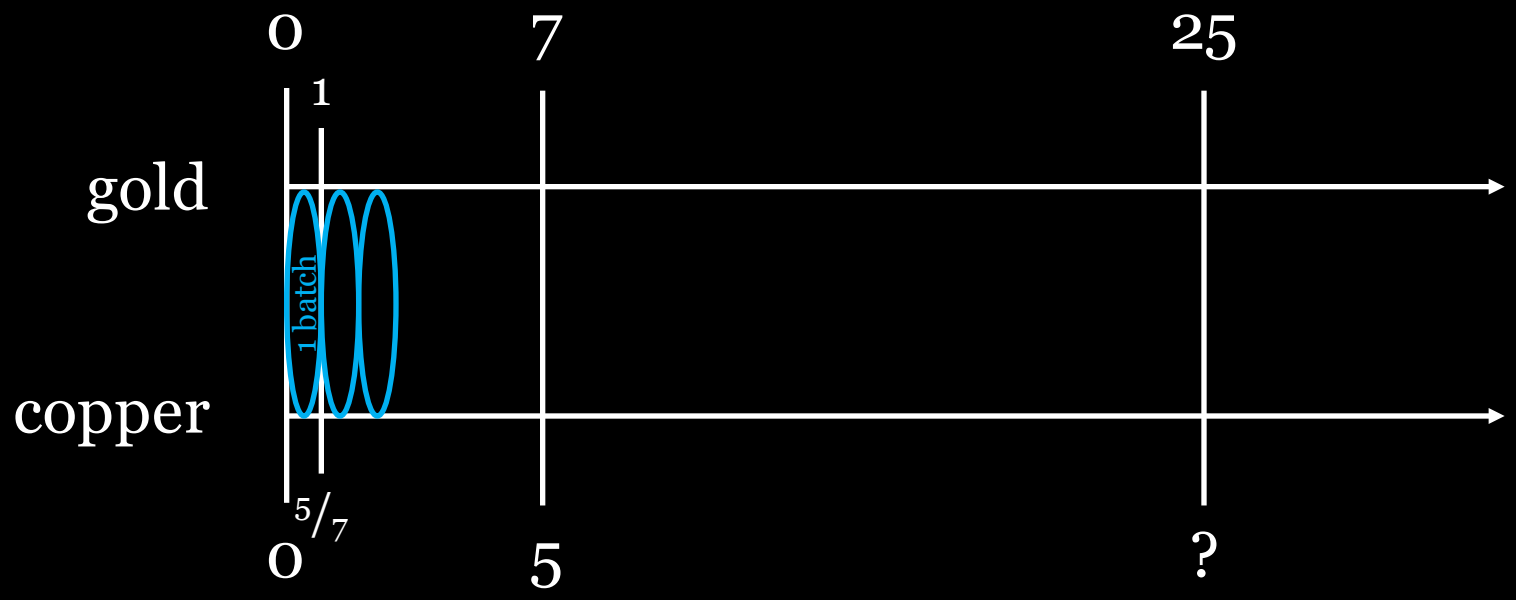

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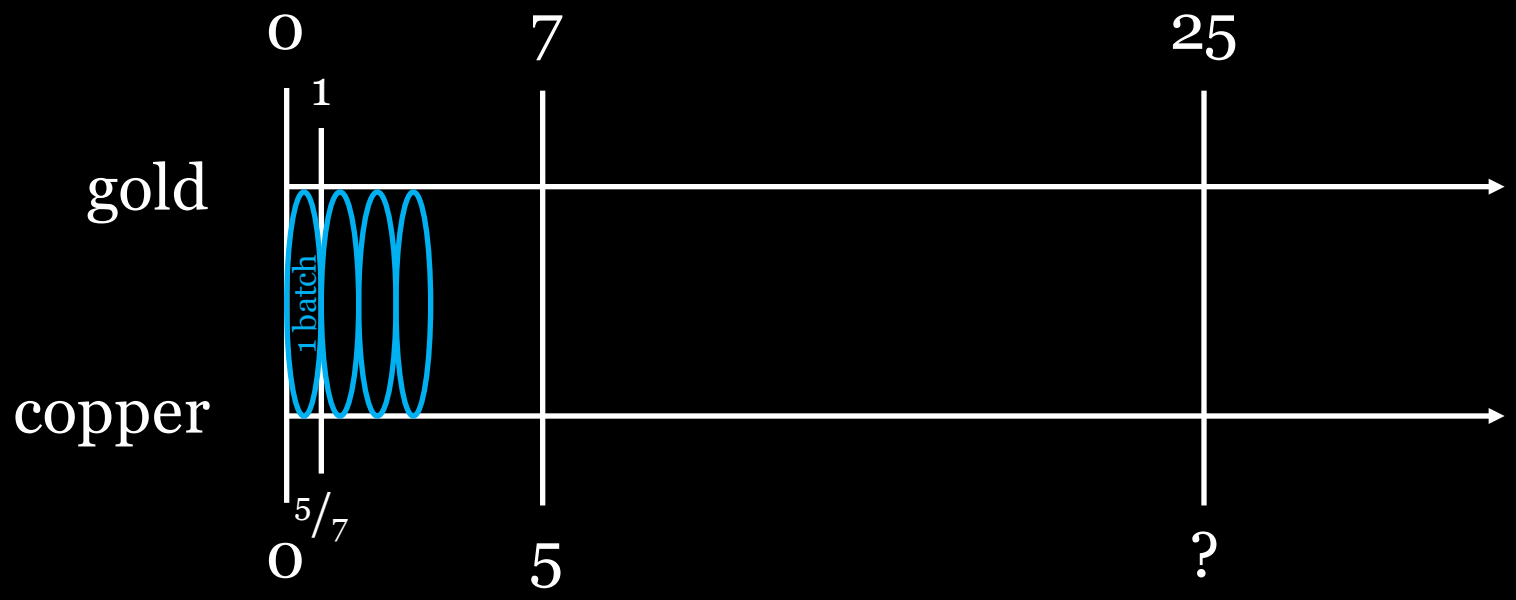

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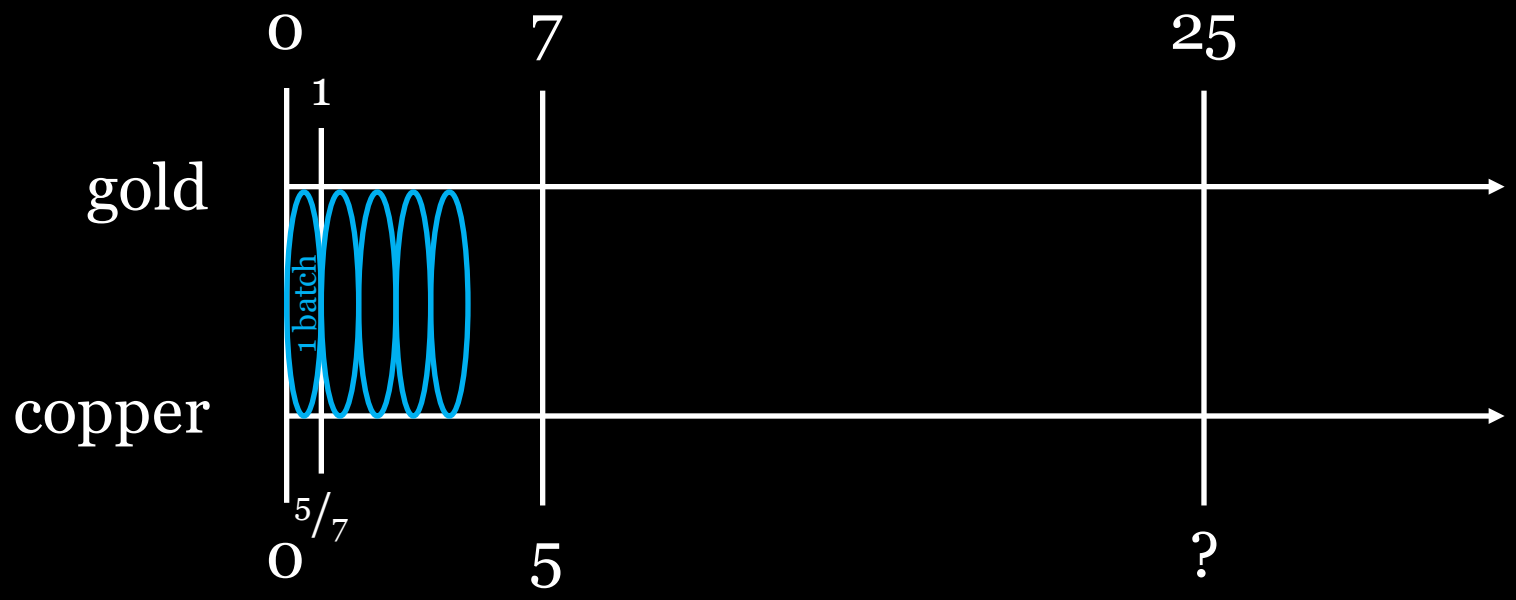

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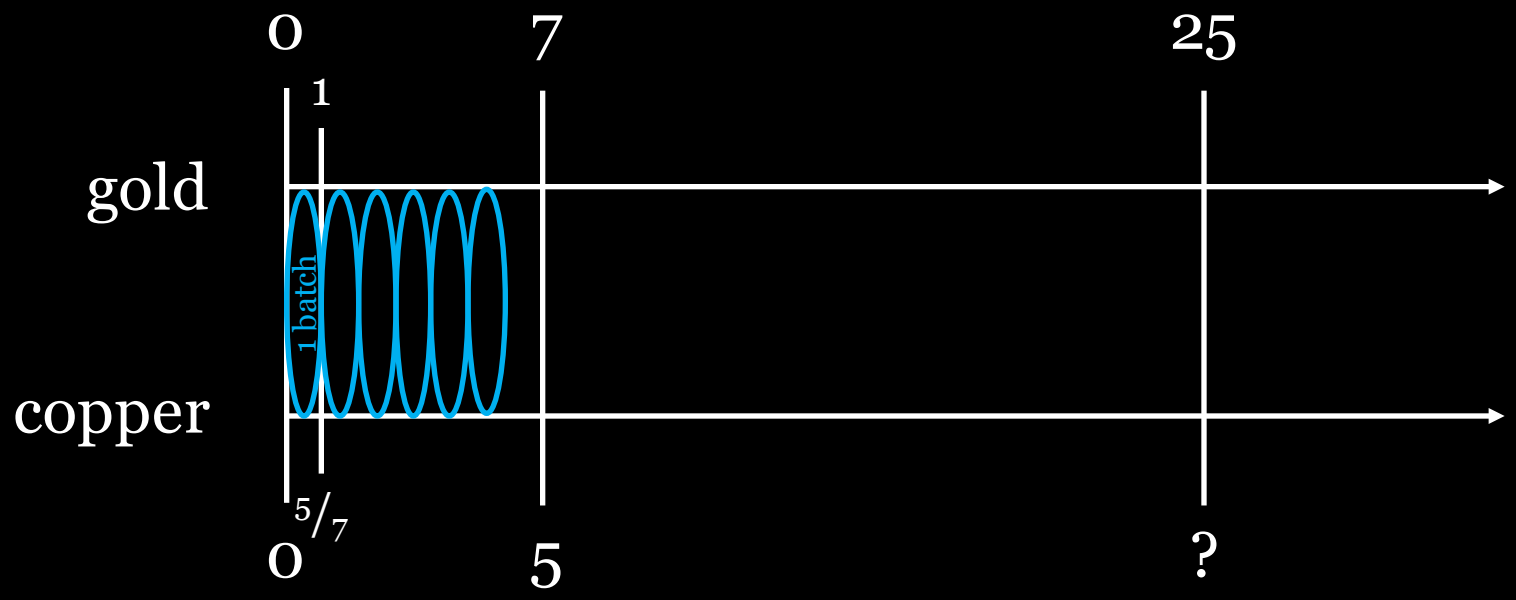

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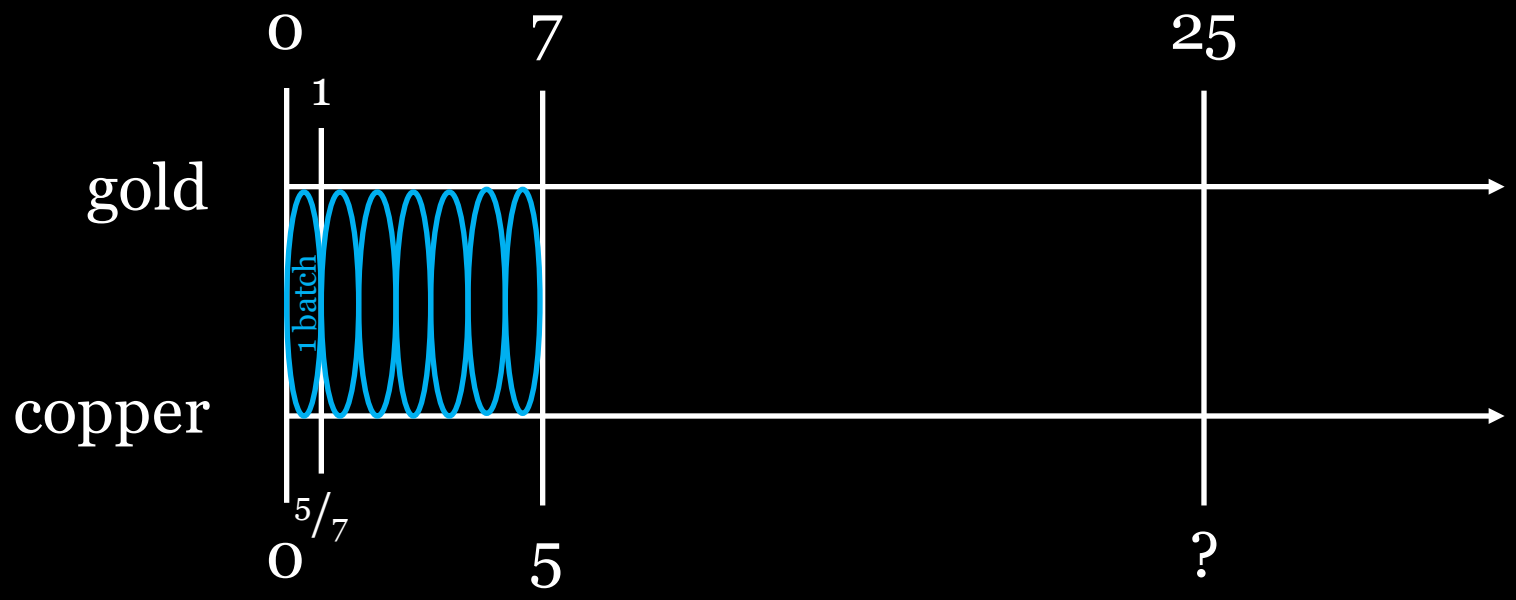

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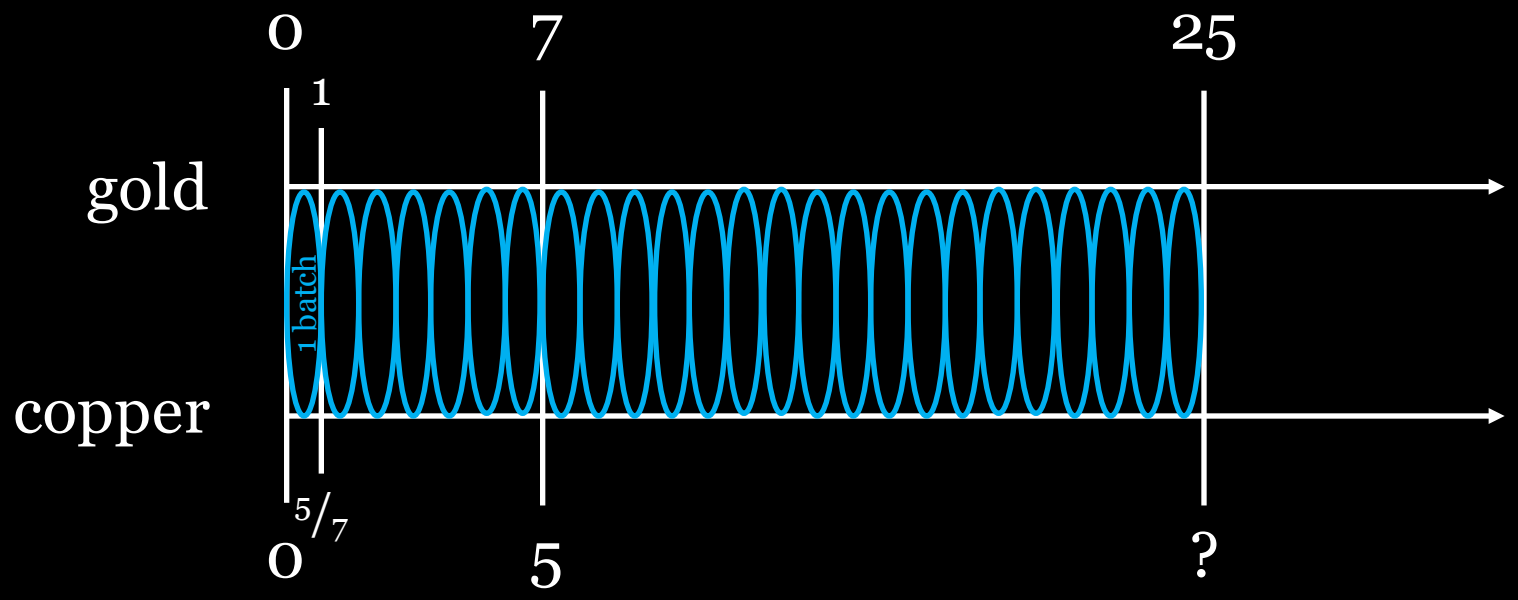

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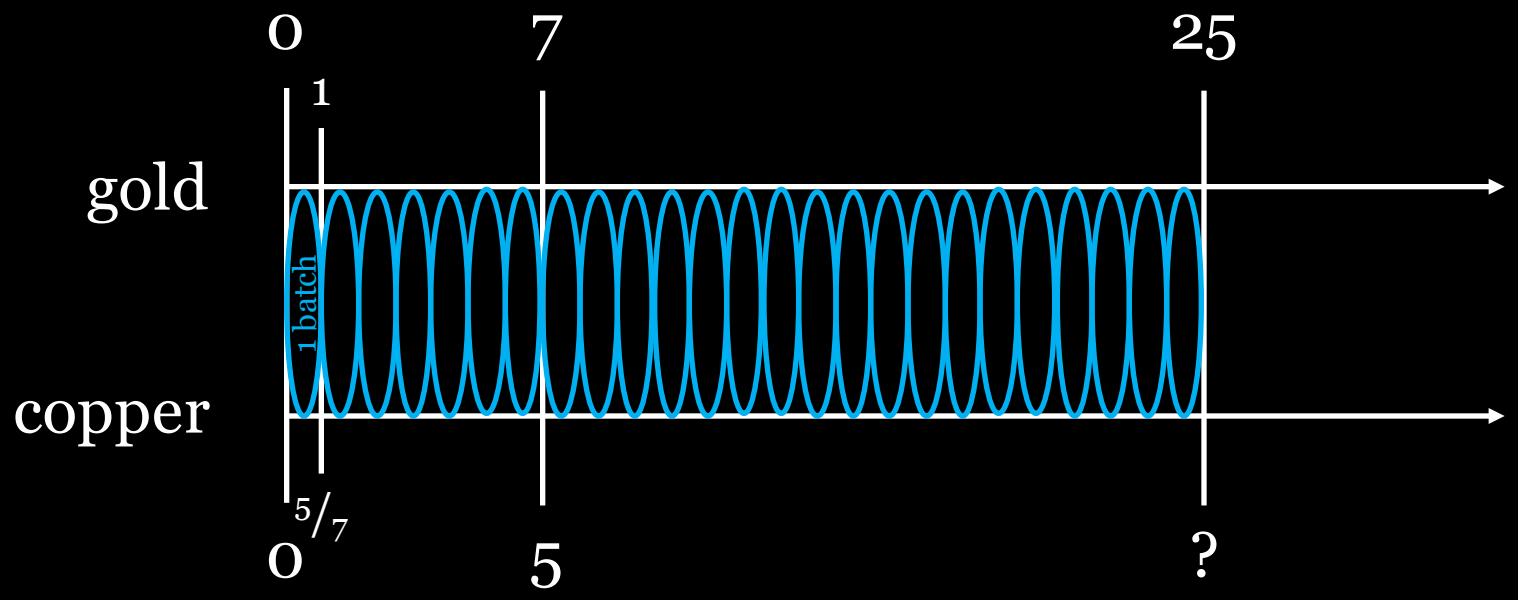
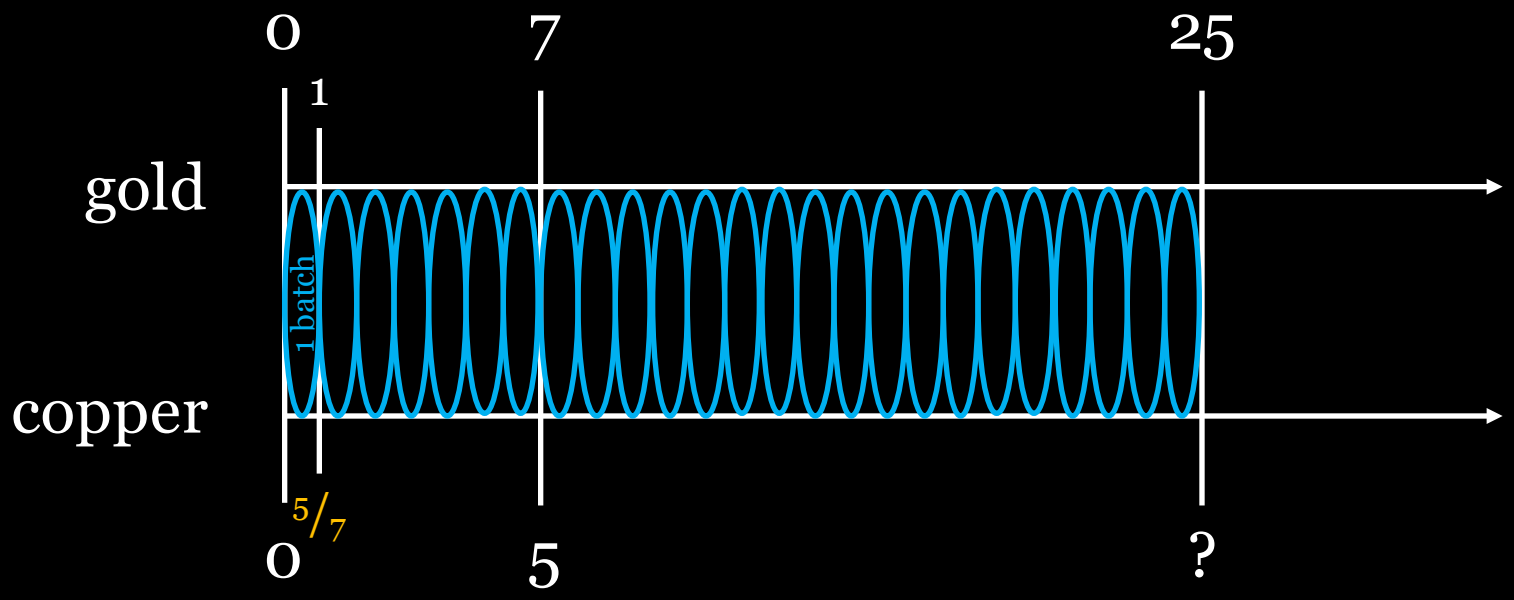


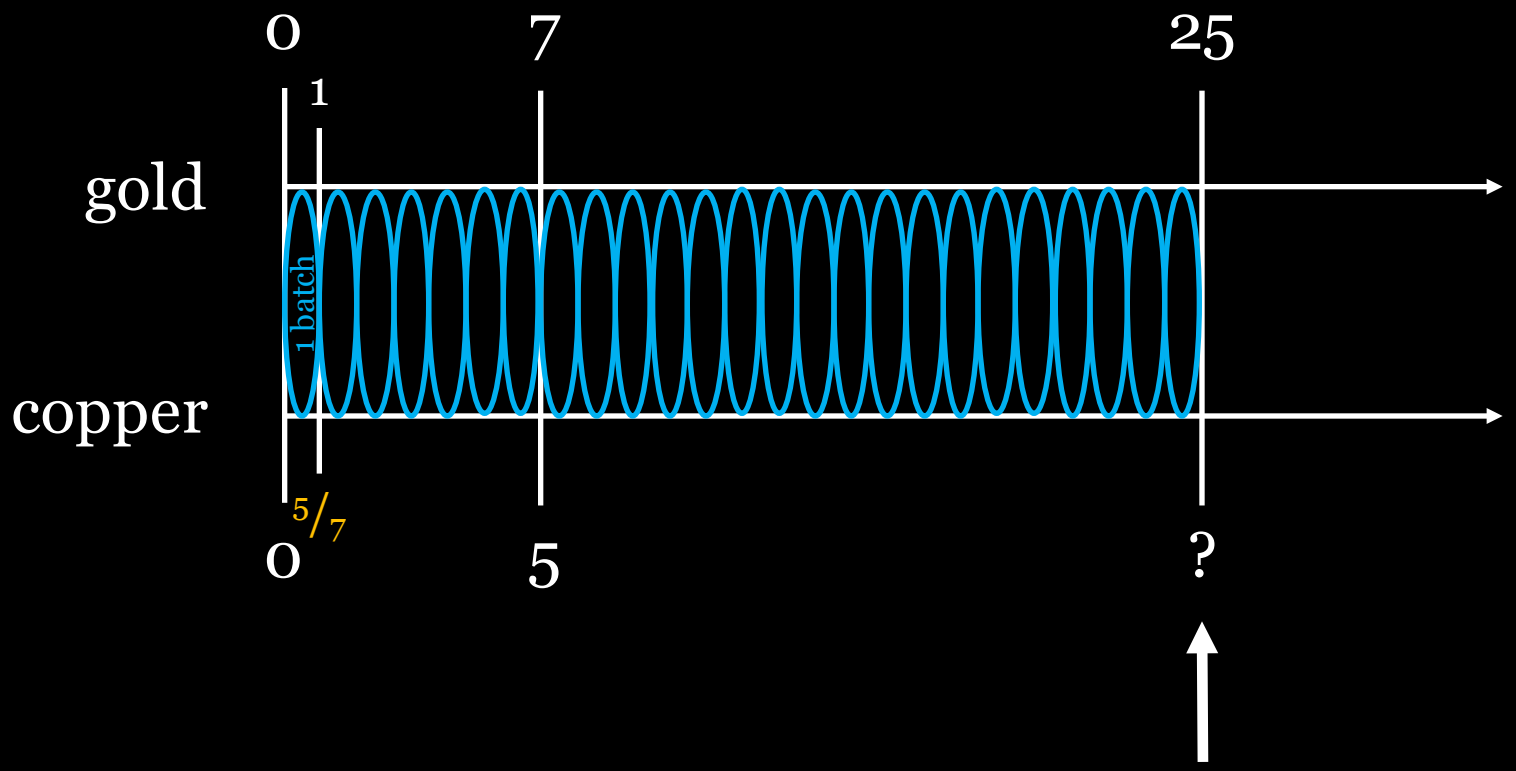
25 batches

Multiply Unit-Rate Batch



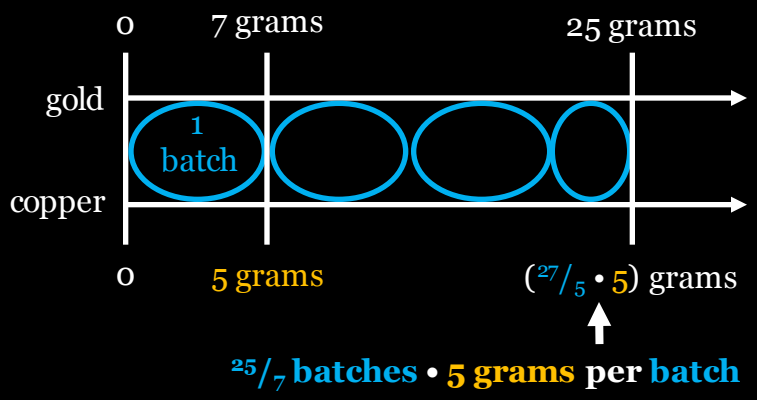
25 batches

Multiply Unit-Rate Batch



25 batches • $^{5}/_{7}$ grams per batch

MULTIPLE BATCHES STRATEGIES

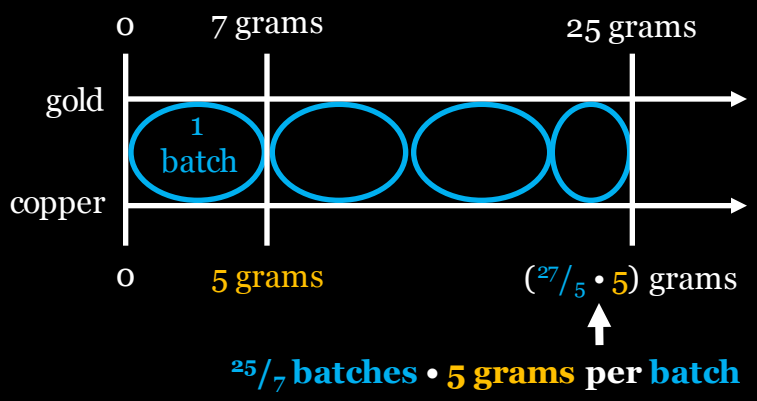


Multiply One Batch

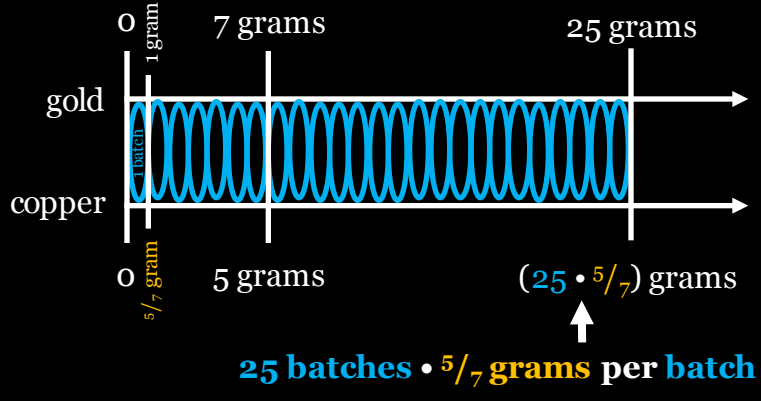
Multiply Unit-Rate Batch

VARIABLE PARTS STRATEGIES

MULTIPLE BATCHES STRATEGIES



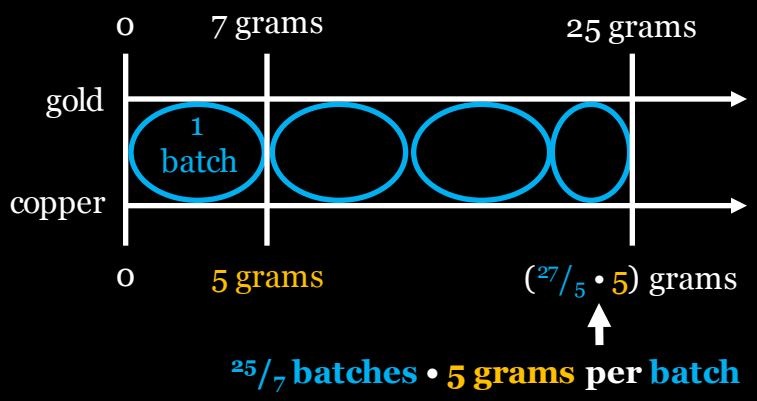
Multiply One Batch



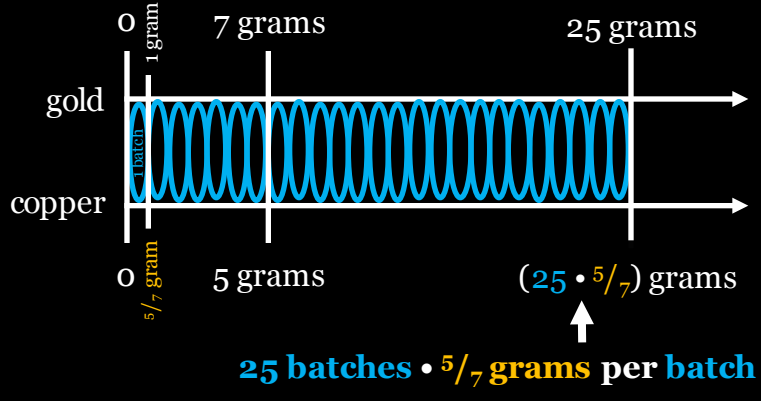
Multiply Unit-Rate Batch

VARIABLE PARTS STRATEGIES

MULTIPLE BATCHES STRATEGIES



Multiply One Batch

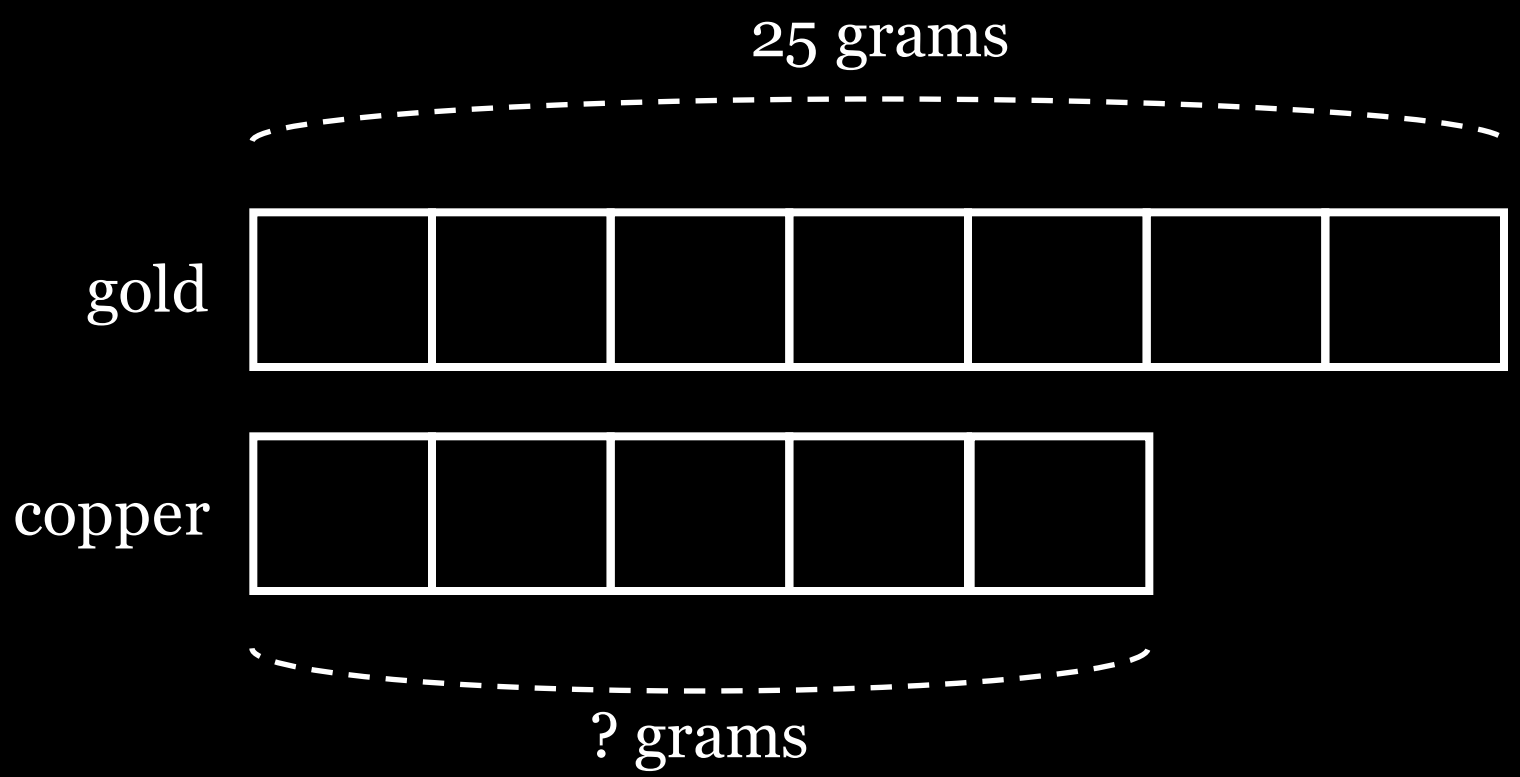


Multiply Unit-Rate Batch

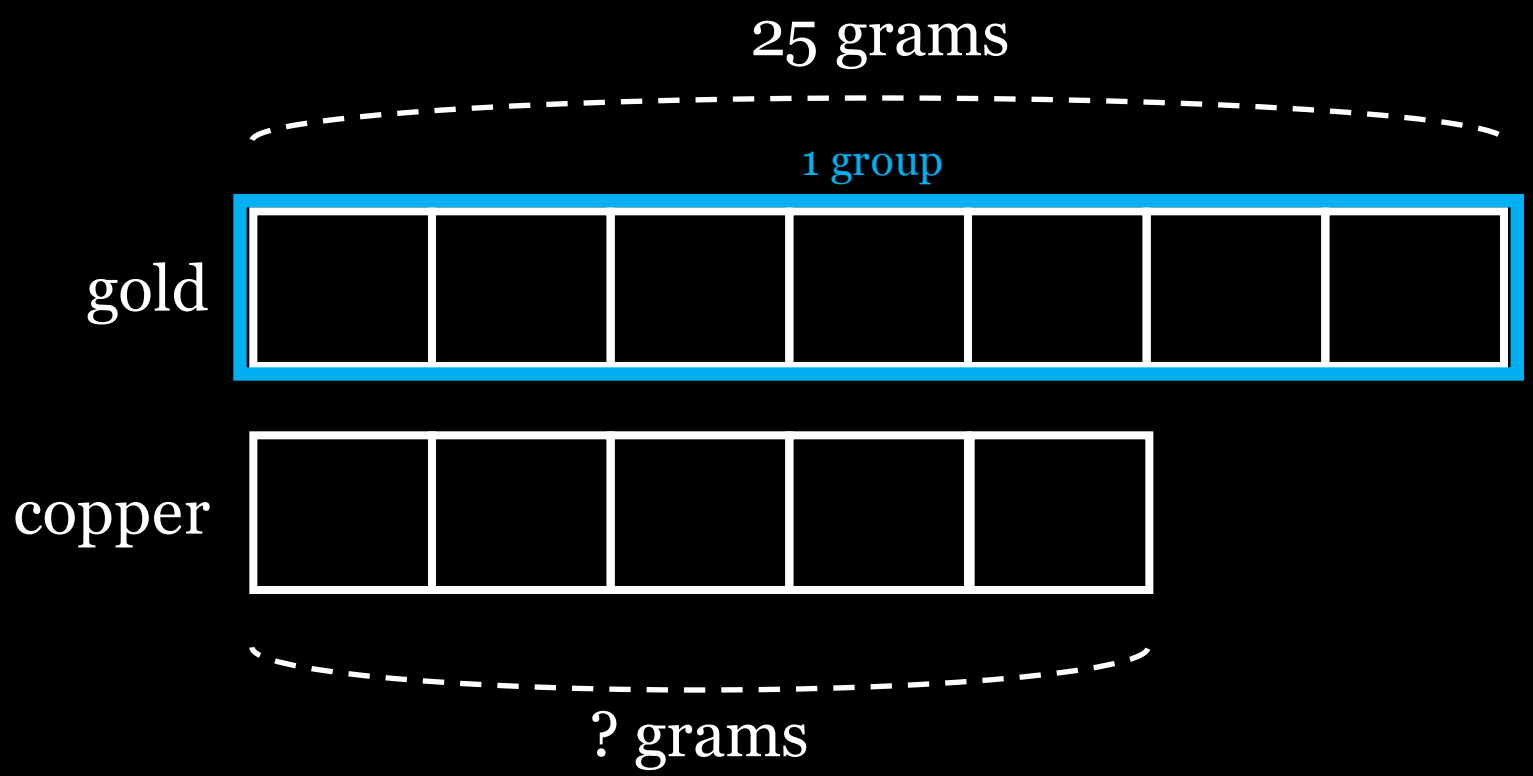
VARIABLE PARTS STRATEGIES

Multiply Total Amount

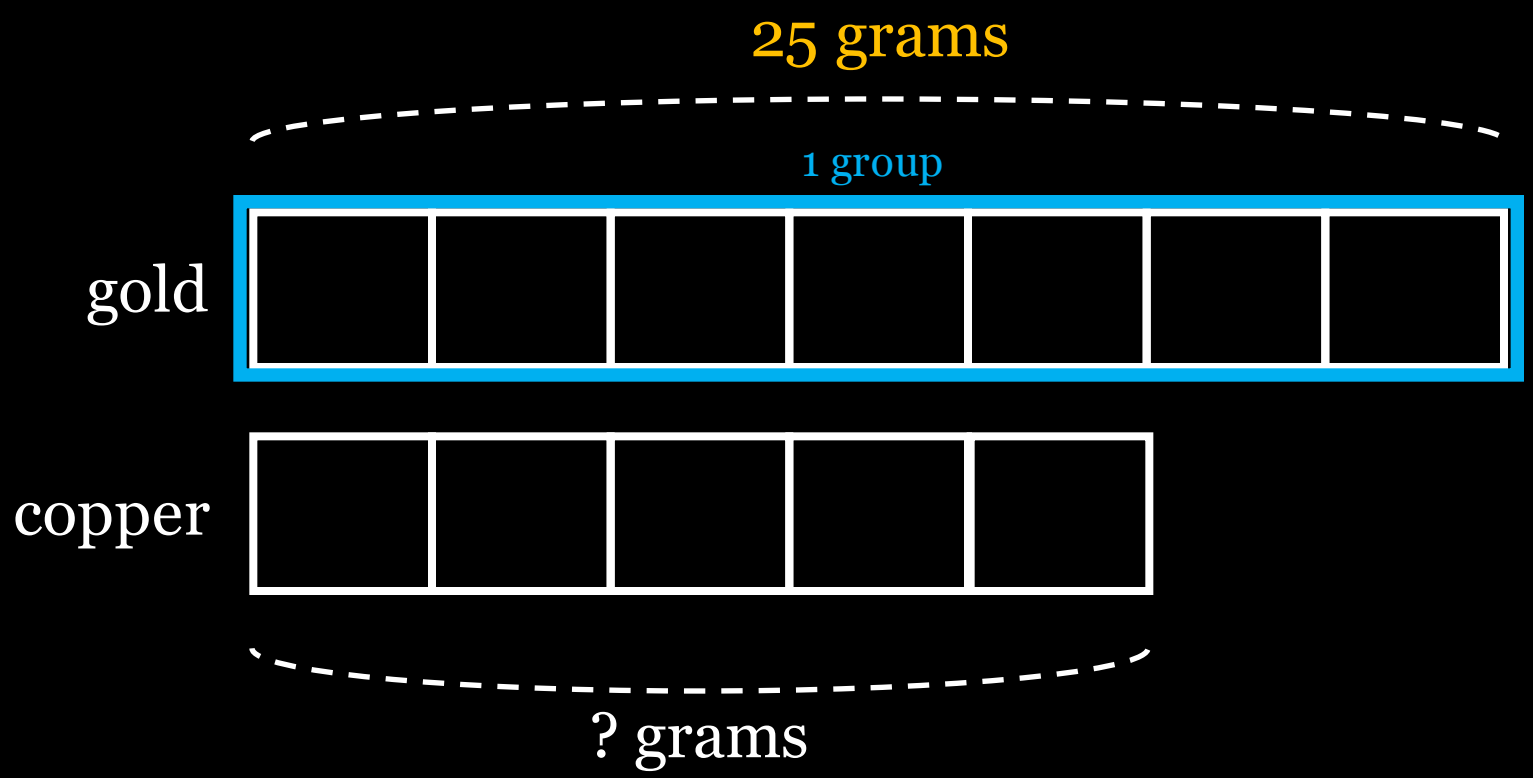
Multiply Total Amount



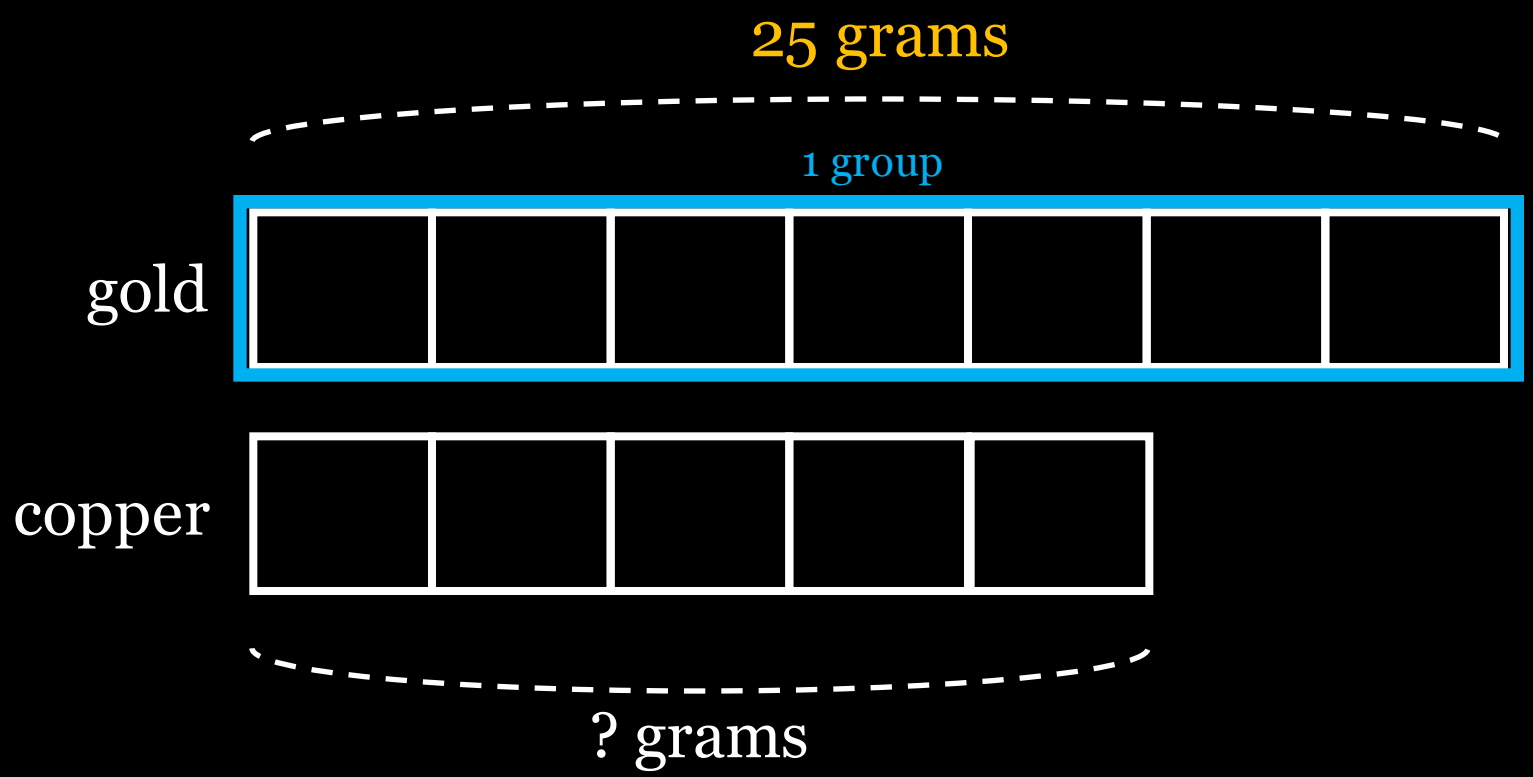
Multiply Total Amount



Multiply Total Amount

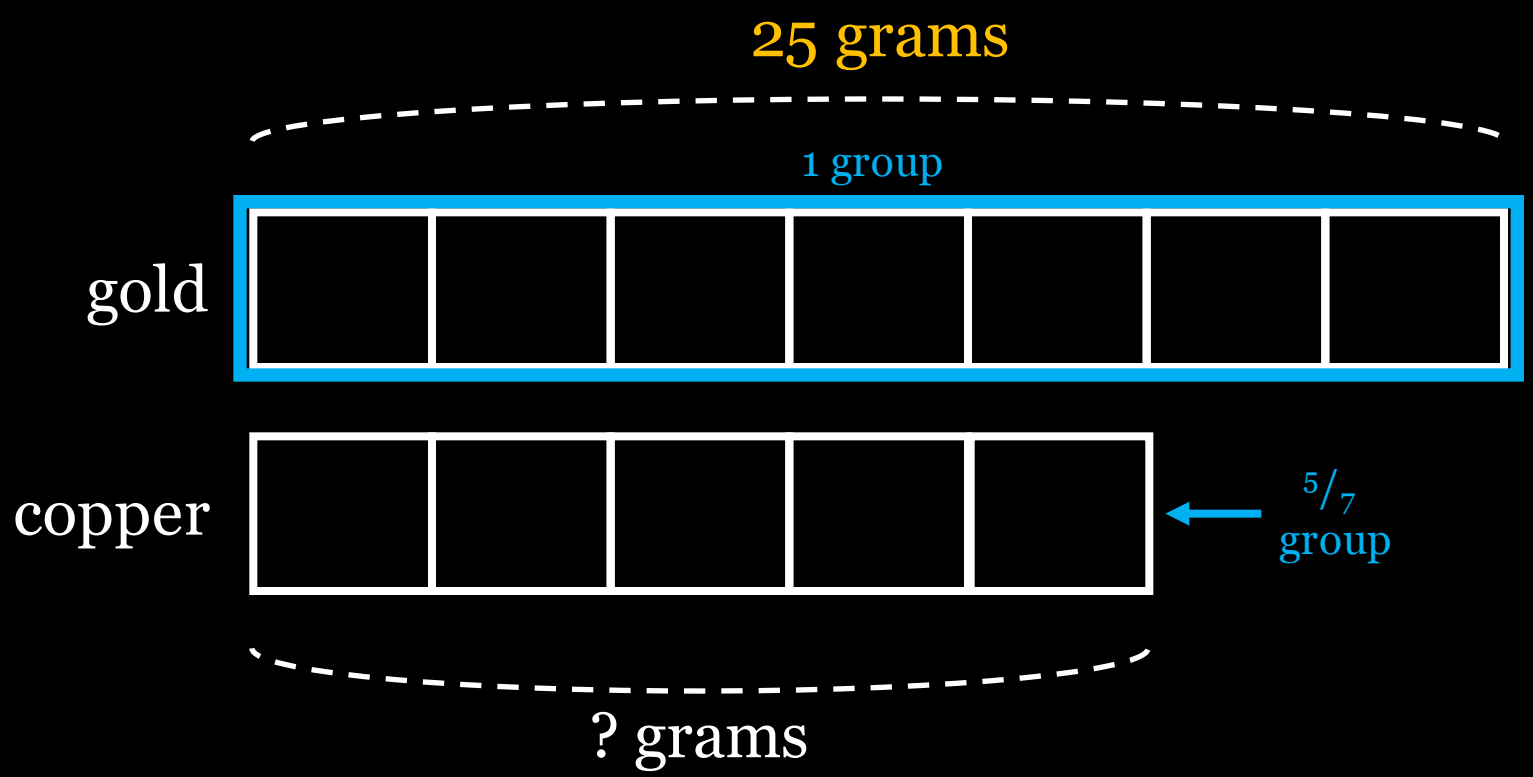


Multiply Total Amount



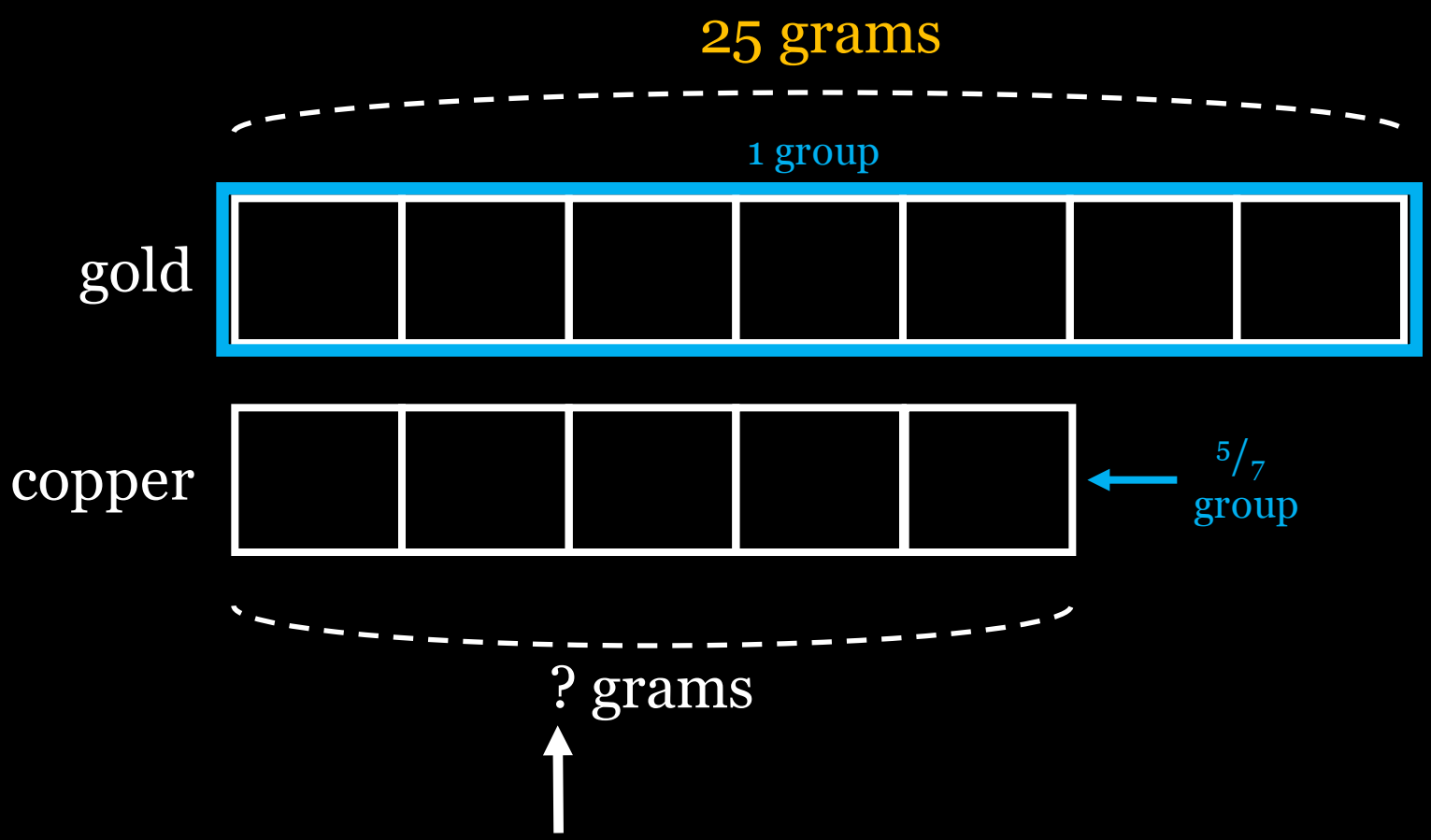
25 grams per group

Multiply Total Amount



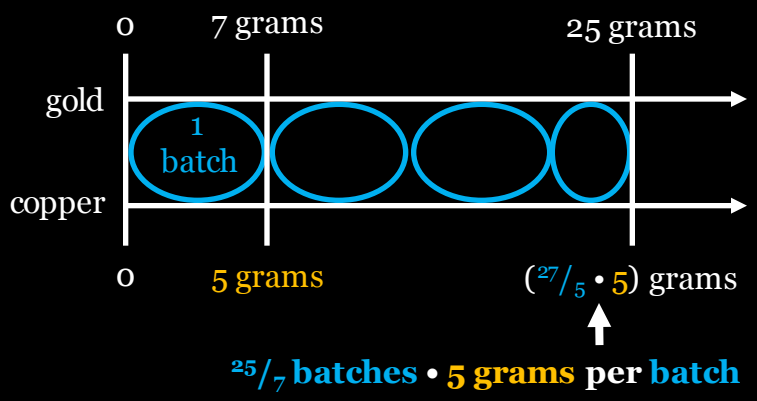
25 grams per group

Multiply Total Amount

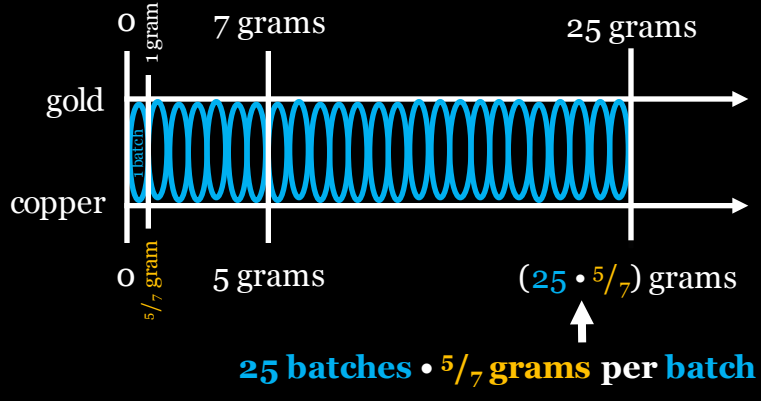


$^{5}/_{7}$ groups • 25 grams per group

MULTIPLE BATCHES STRATEGIES



Multiply One Batch

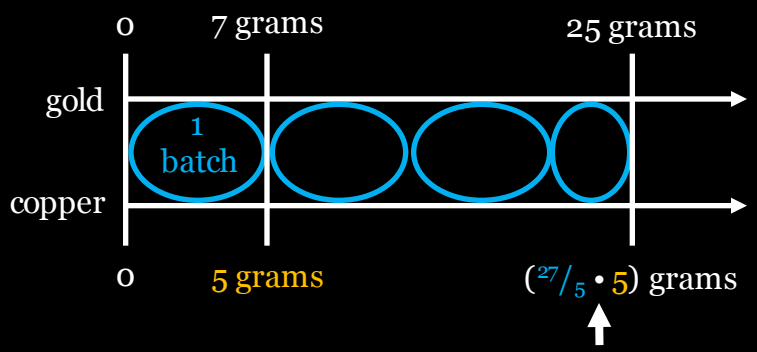


Multiply Unit-Rate Batch

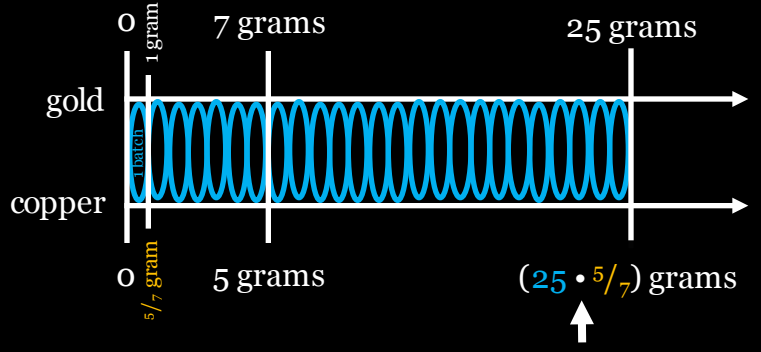
VARIABLE PARTS STRATEGIES

Multiply Total Amount

MULTIPLE BATCHES STRATEGIES

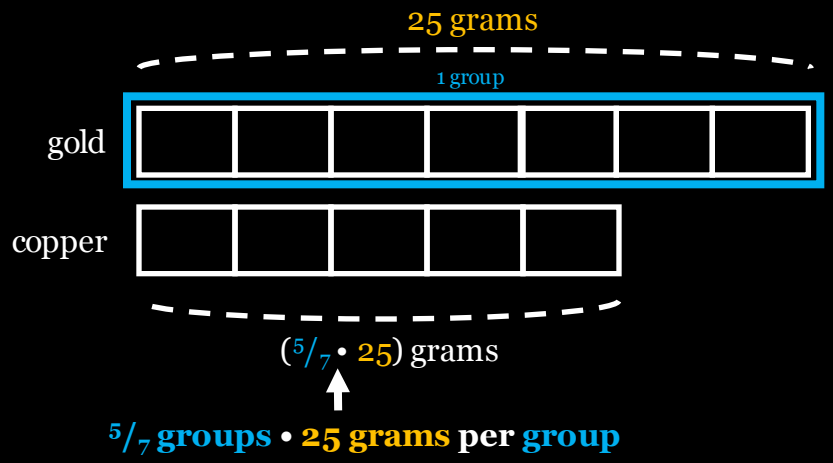


Multiply One Batch



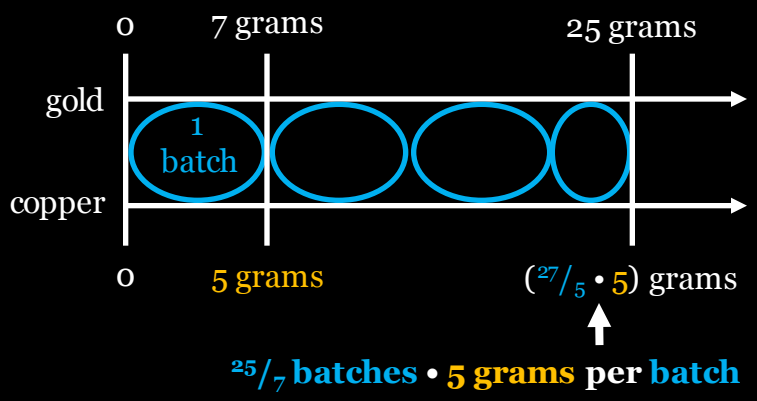
Multiply Unit-Rate Batch

VARIABLE PARTS STRATEGIES

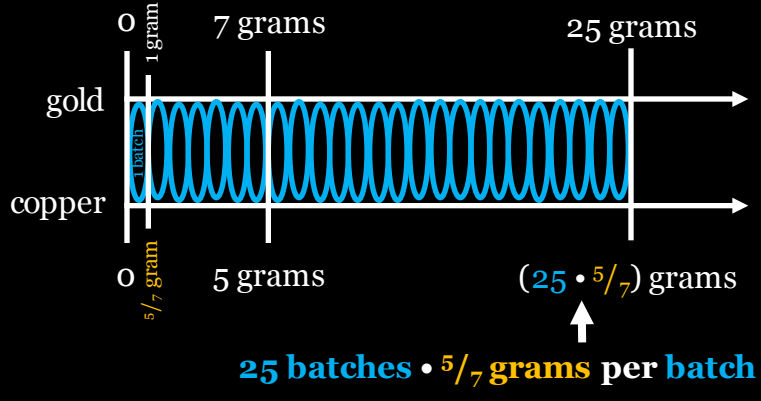


Multiply Total Amount

MULTIPLE BATCHES STRATEGIES

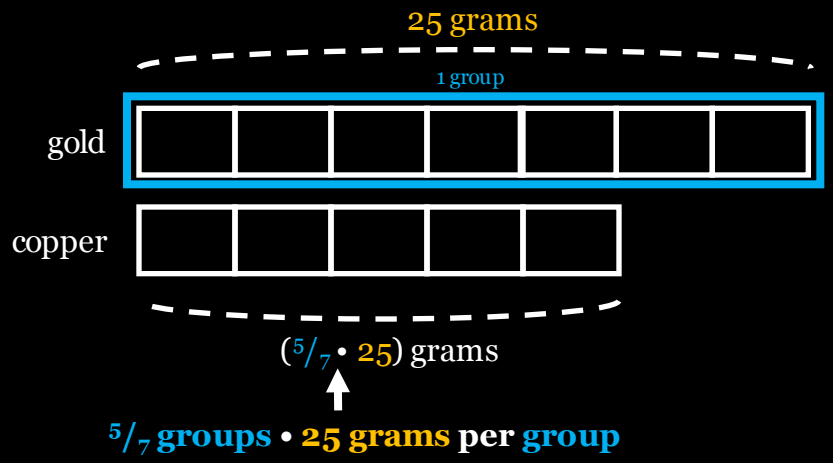


Multiply One Batch



Multiply Unit-Rate Batch

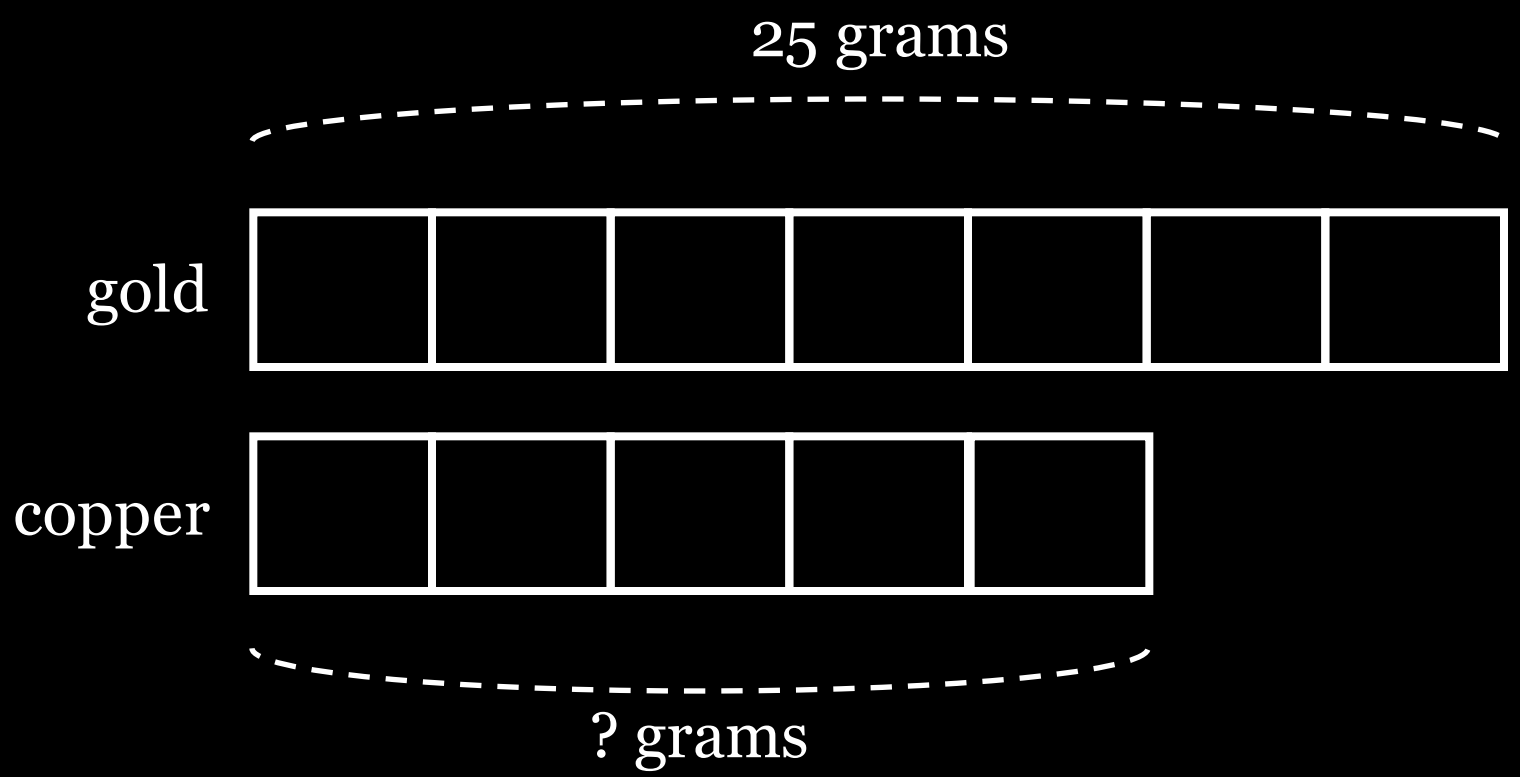
VARIABLE PARTS STRATEGIES



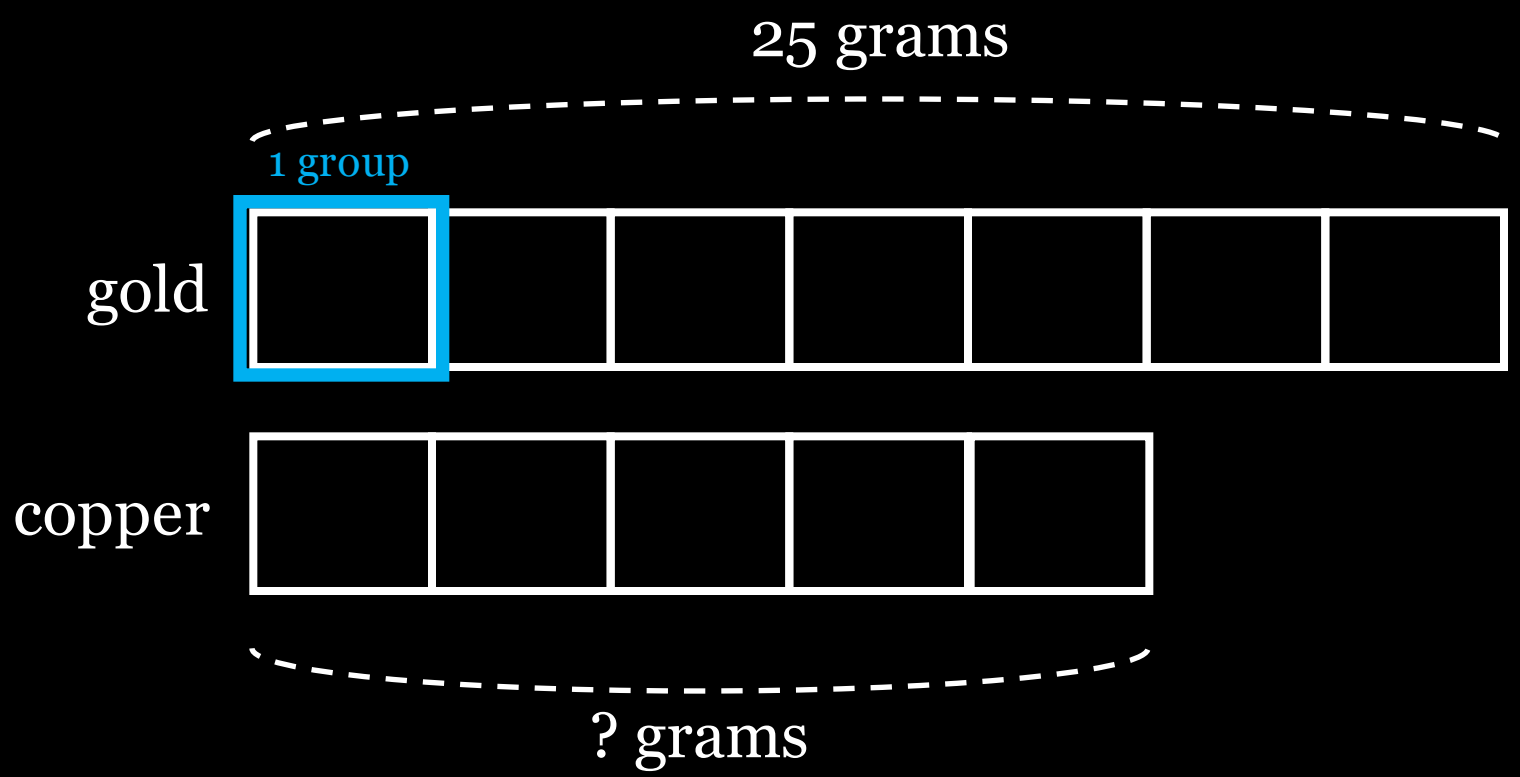
Multiply Total Amount

Multiply One Part

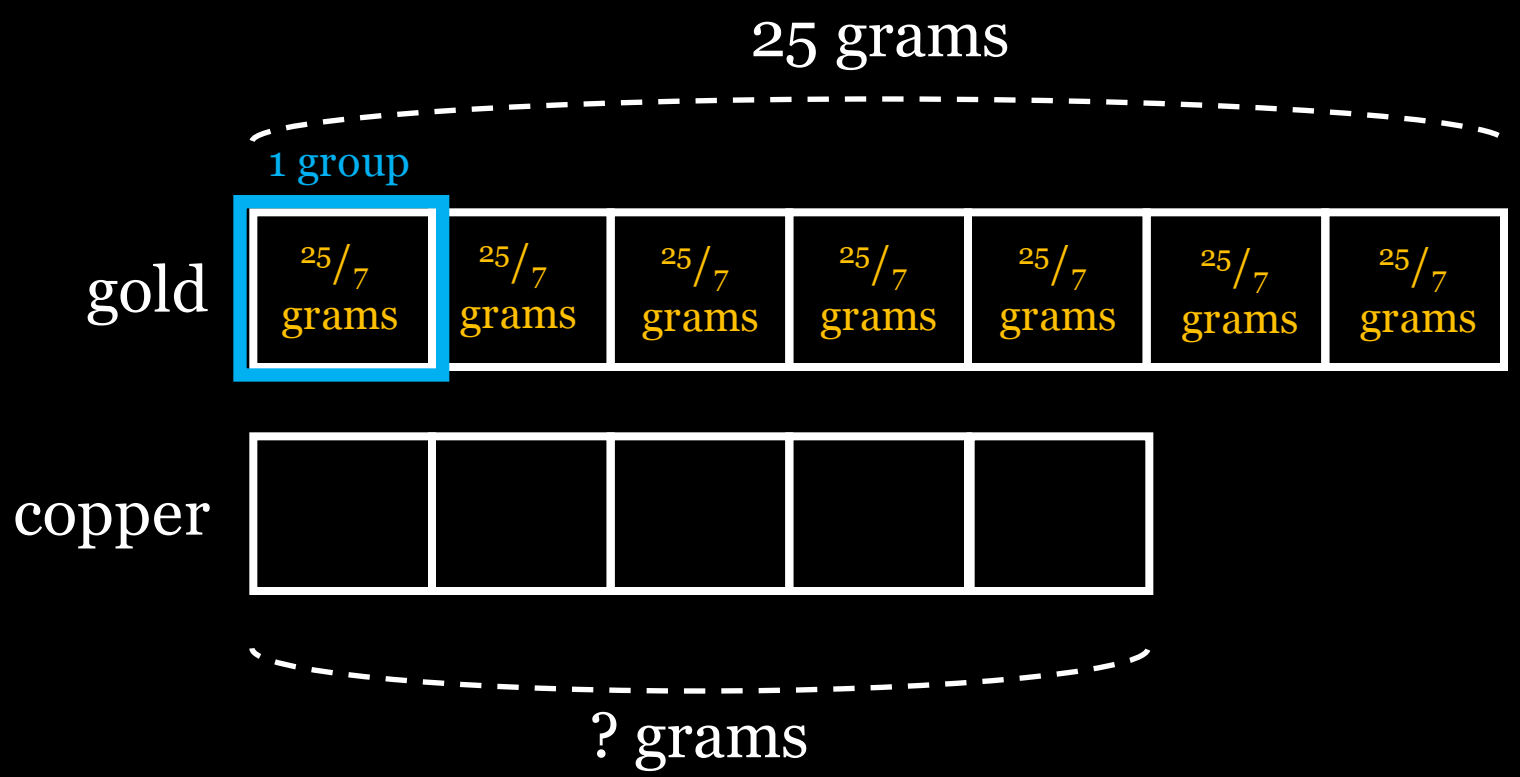
Multiply One Part



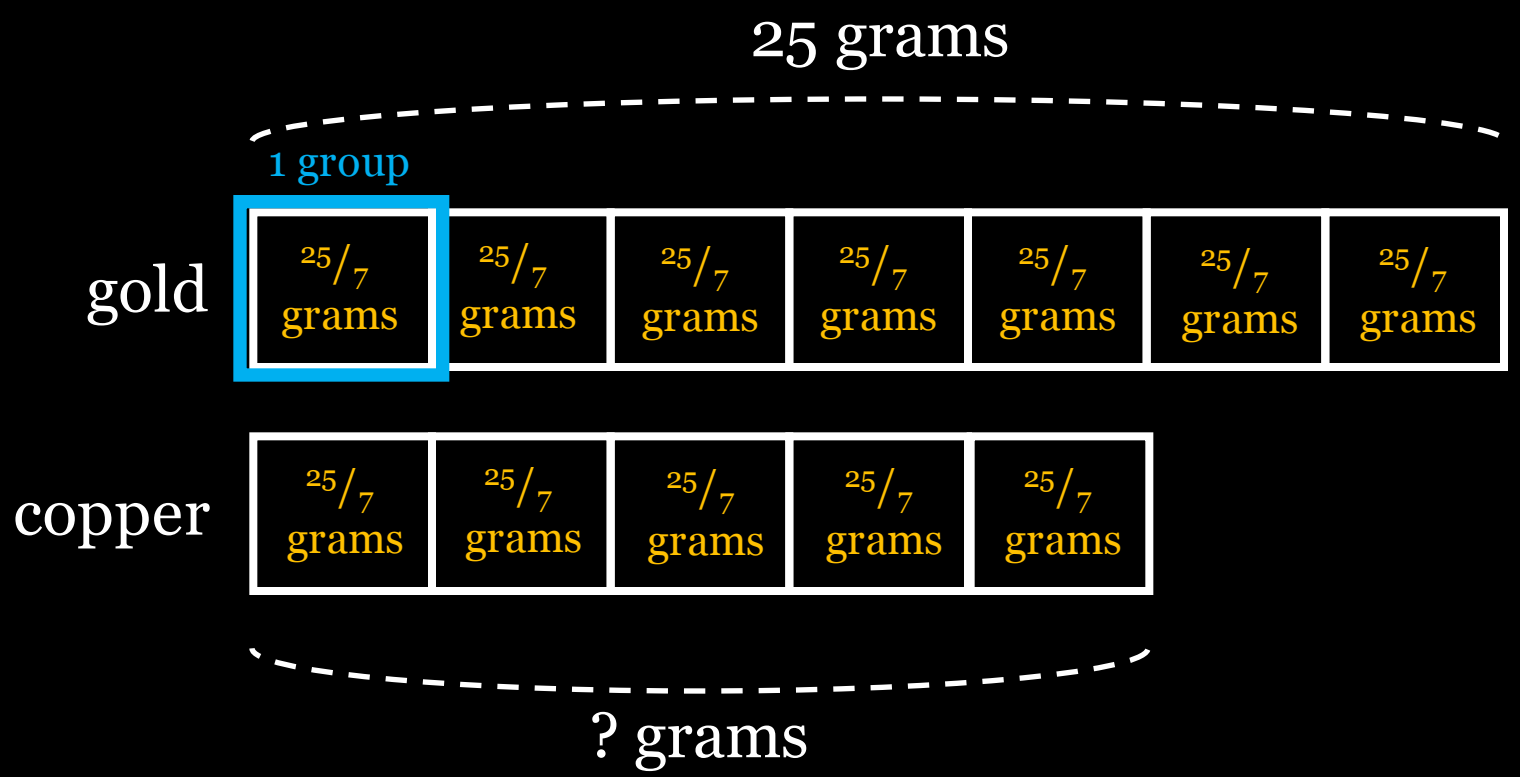
Multiply One Part



Multiply One Part

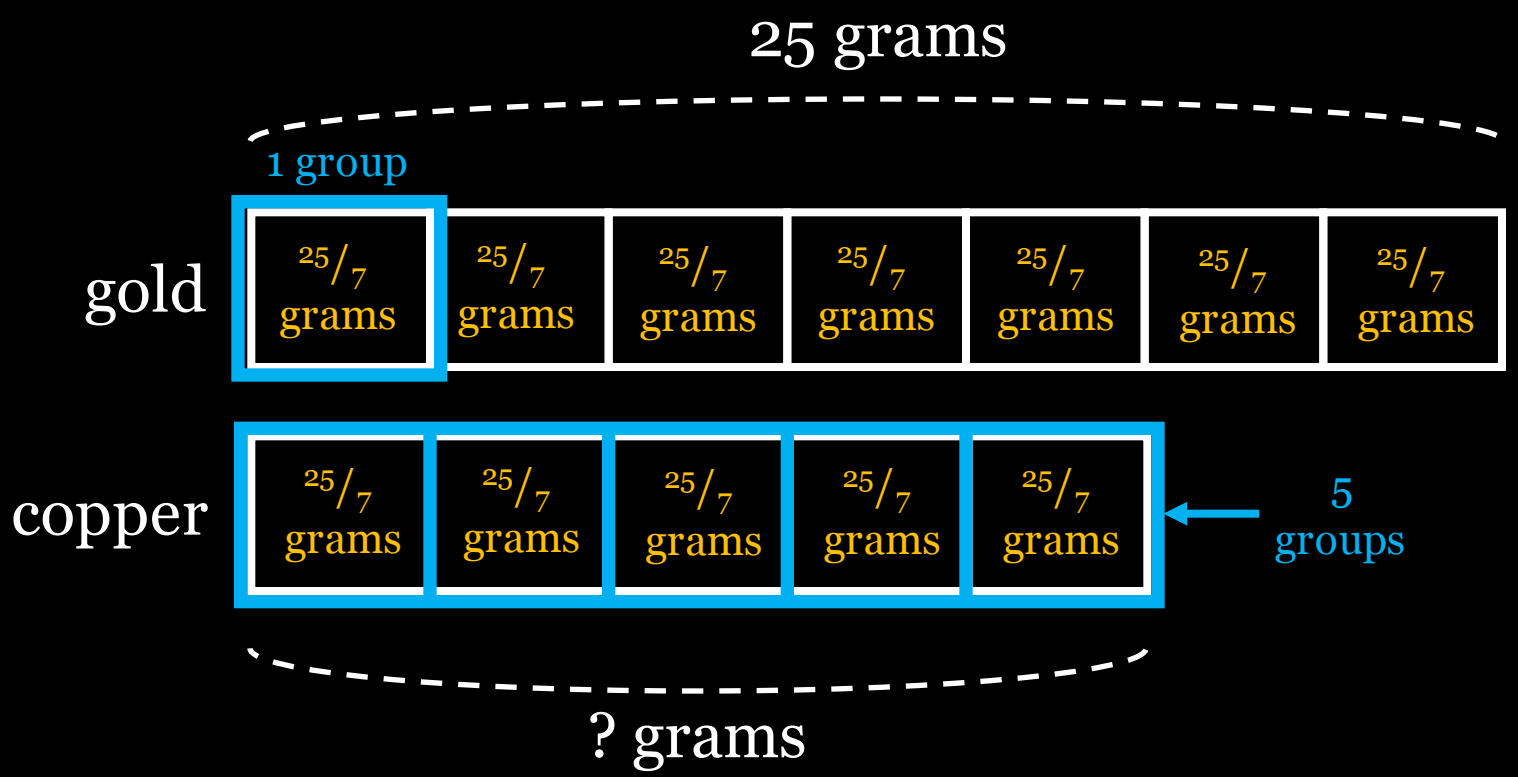


Multiply One Part



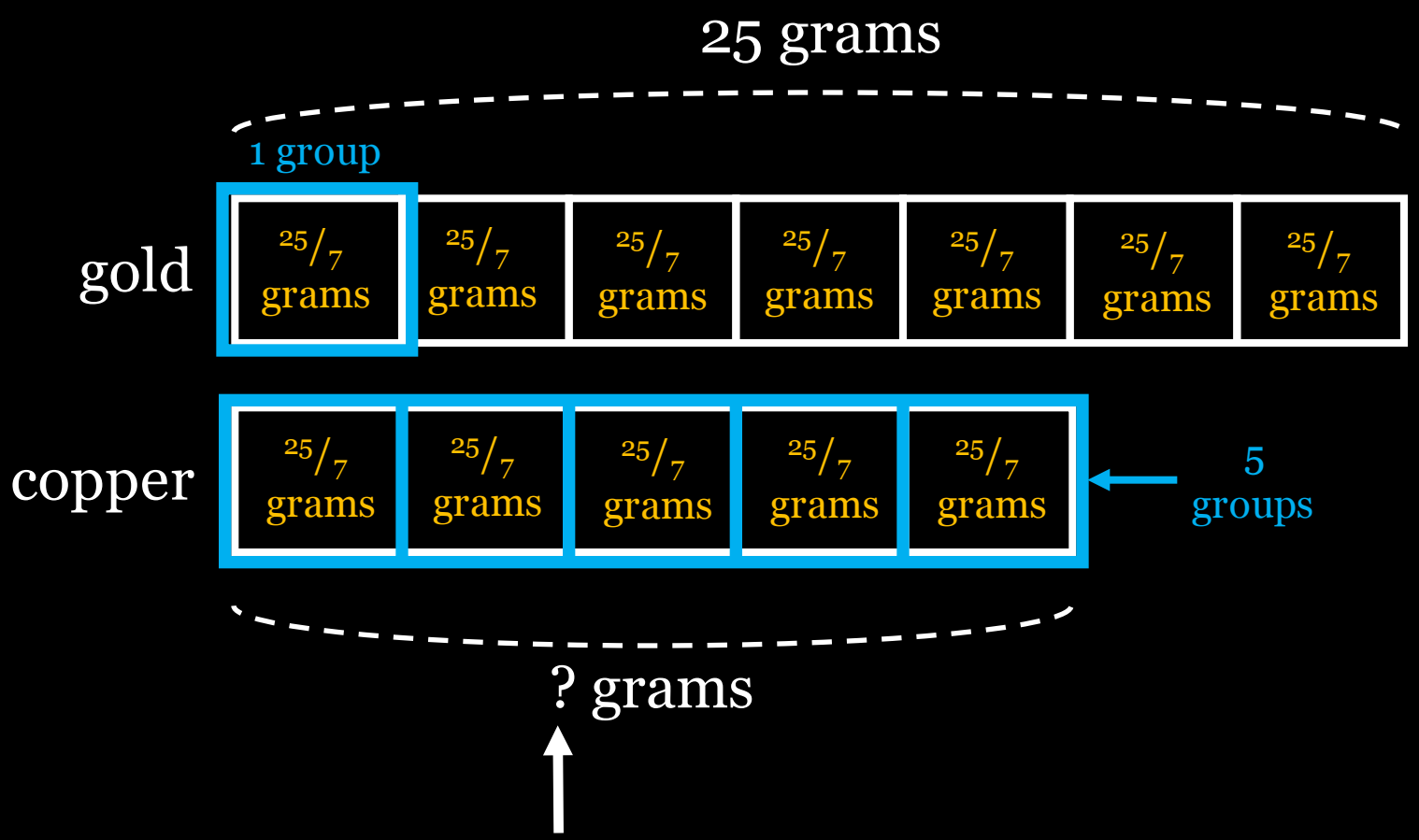
$^{25}/_{7}$ grams per group

Multiply One Part



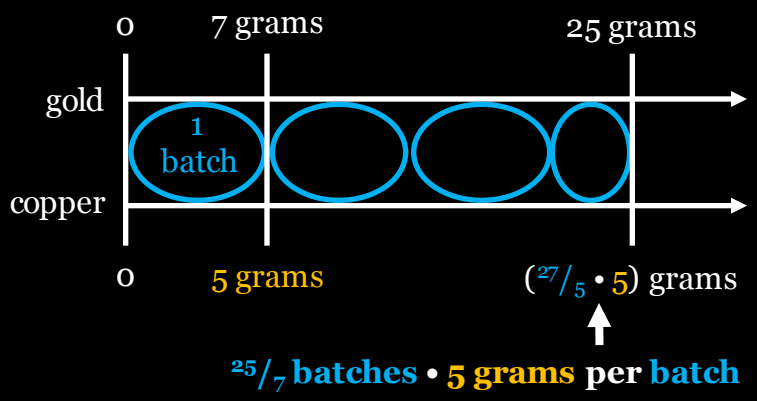
$^{25}/_{7}$ grams per group

Multiply One Part

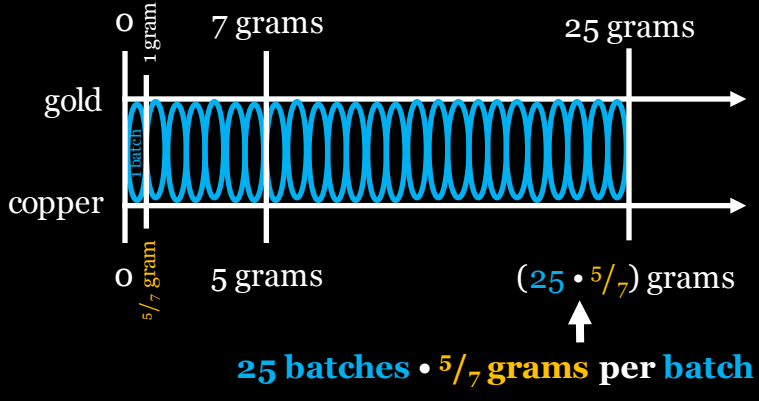


5 groups • $^{25}/_{7}$ grams per group

MULTIPLE BATCHES STRATEGIES

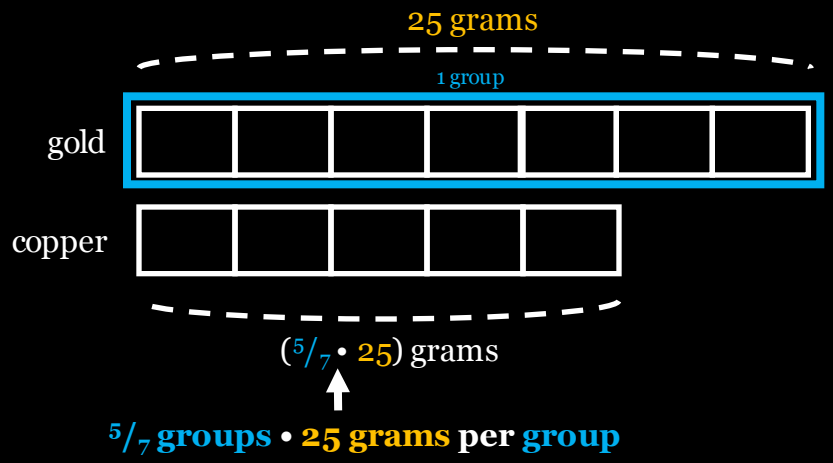


Multiply One Batch



Multiply Unit-Rate Batch

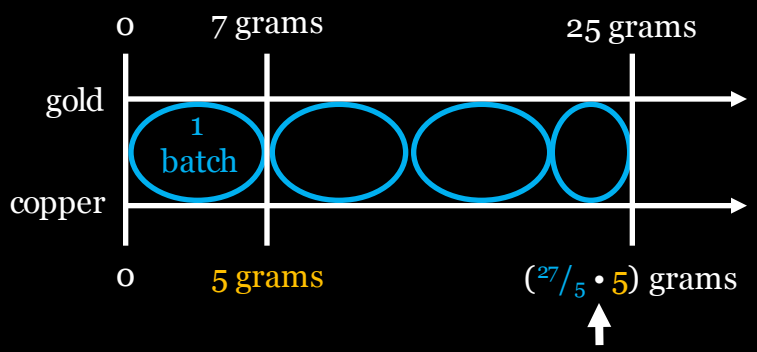
VARIABLE PARTS STRATEGIES



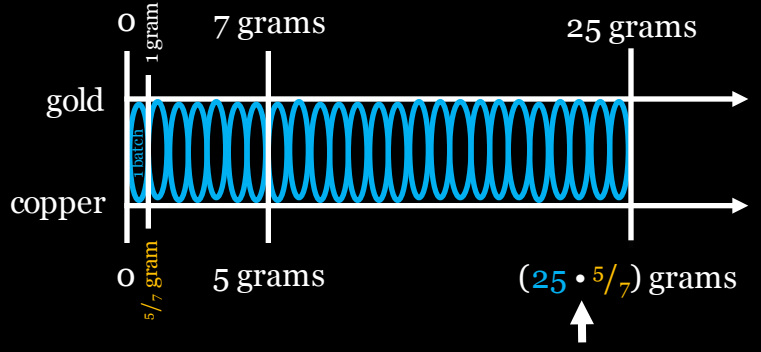
Multiply Total Amount

Multiply One Part

MULTIPLE BATCHES STRATEGIES

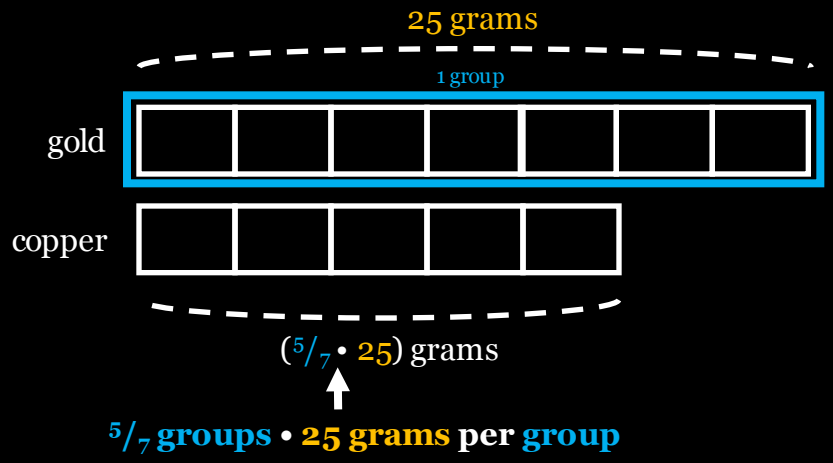


Multiply One Batch

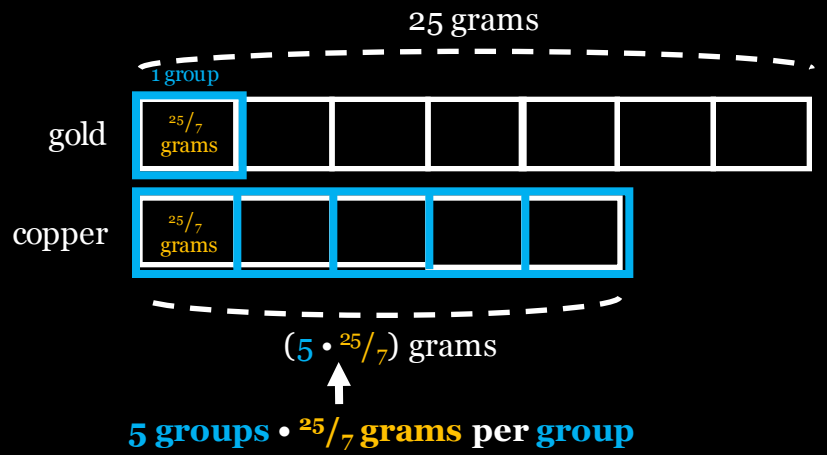


Multiply Unit-Rate Batch

VARIABLE PARTS STRATEGIES



Multiply Total Amount



Multiply One Part

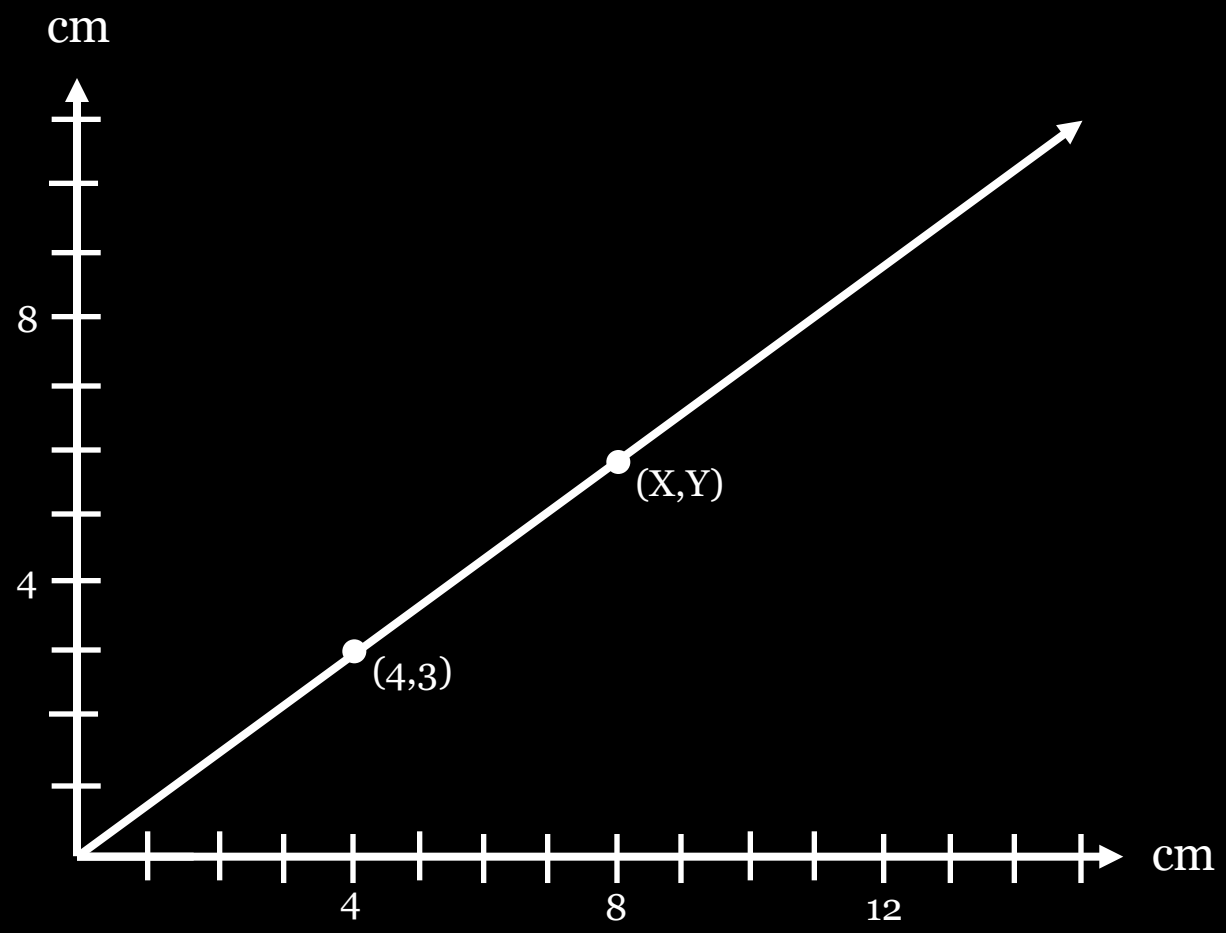
THOUGHTS?!

EXTENSIONS USING THE VARIABLE PARTS PERSPECTIVE

GENERATE AN EQUATION FOR A LINE

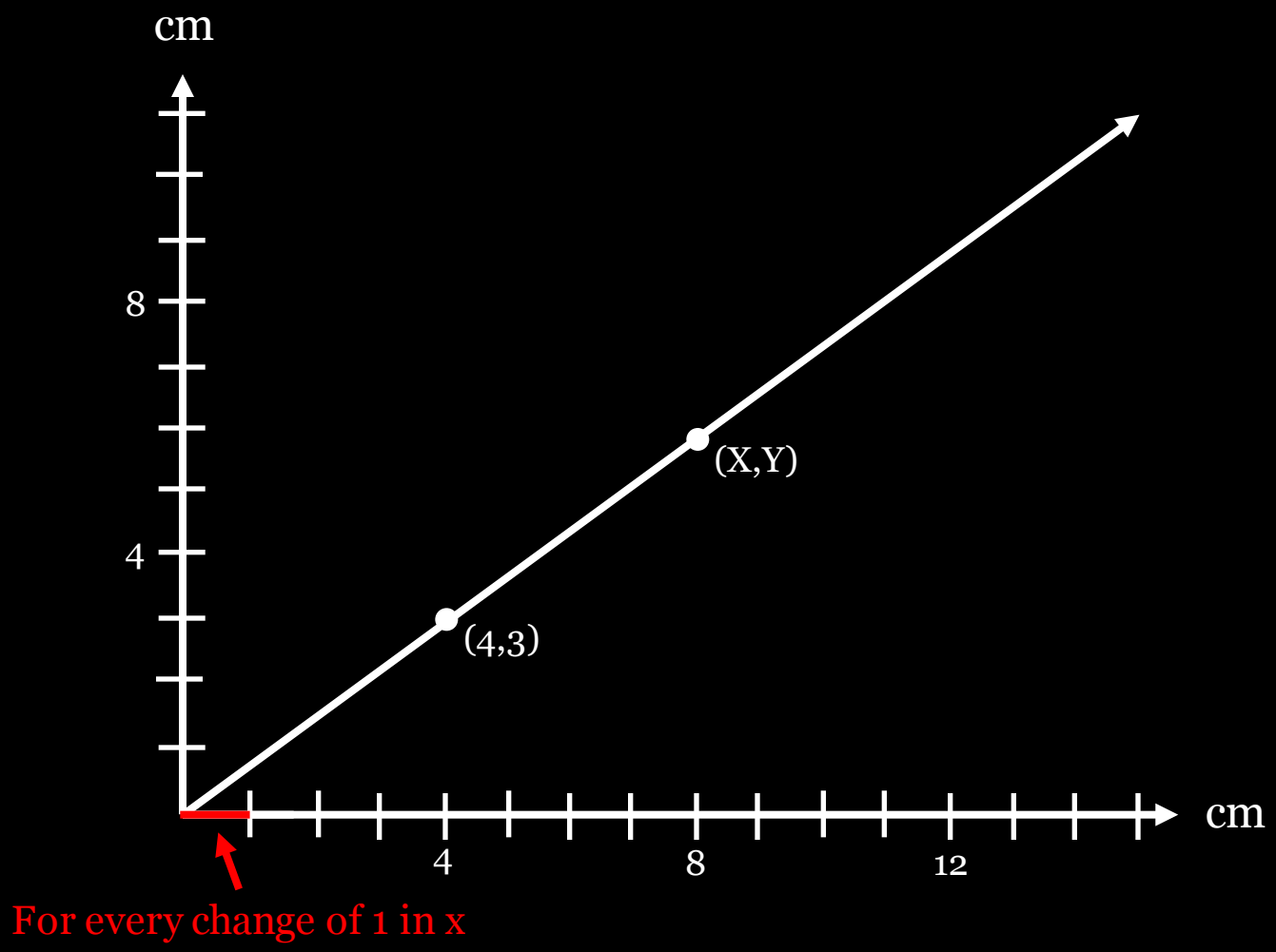
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



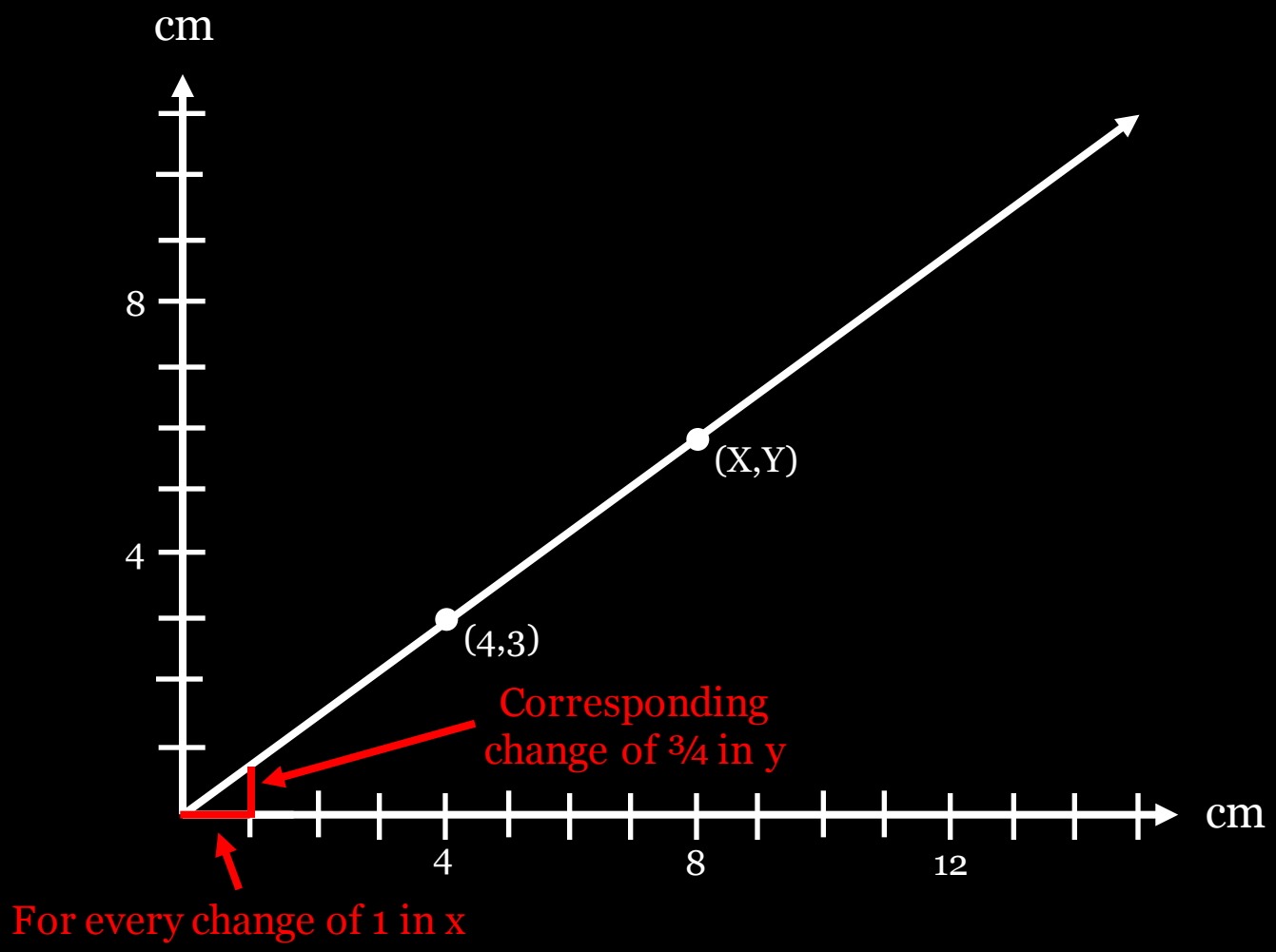
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



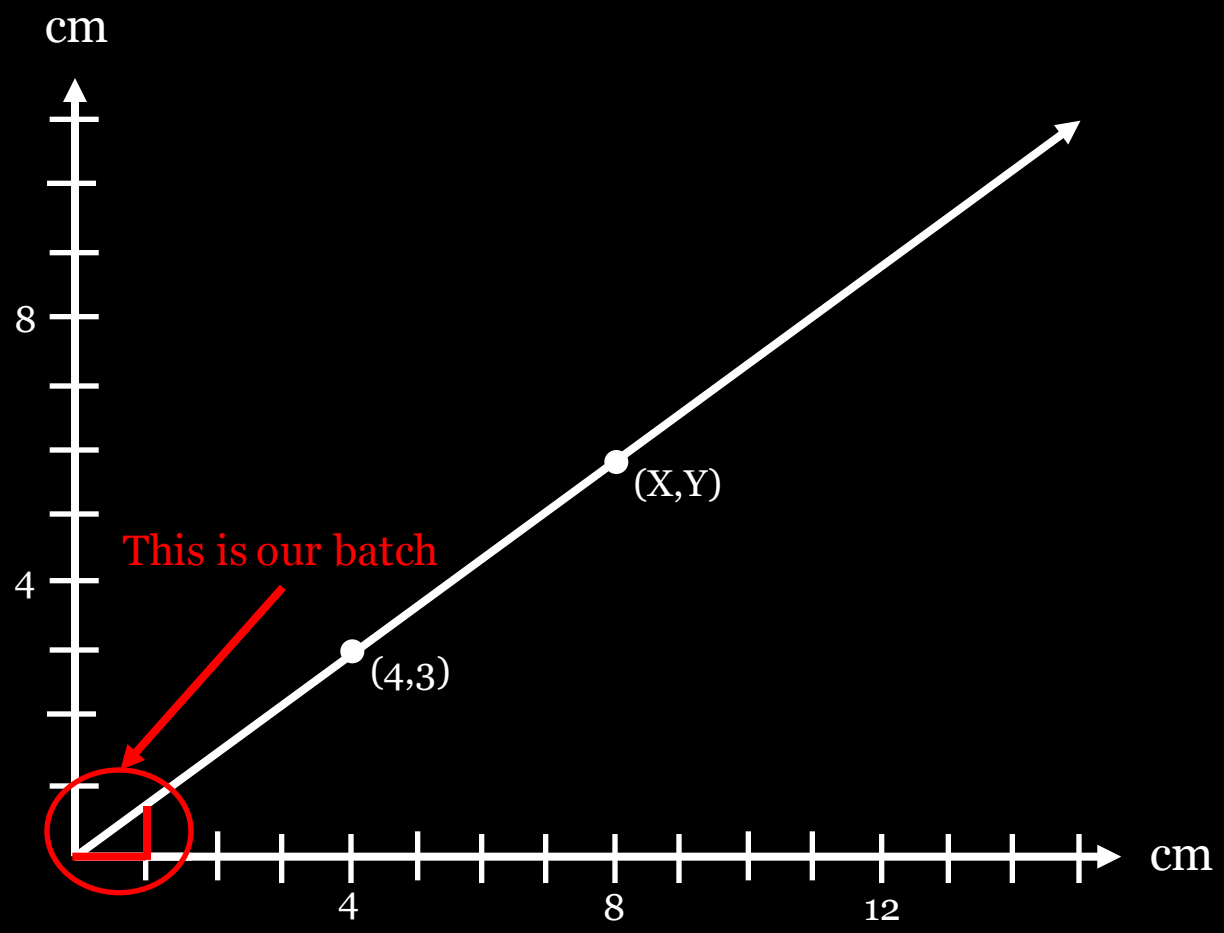
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



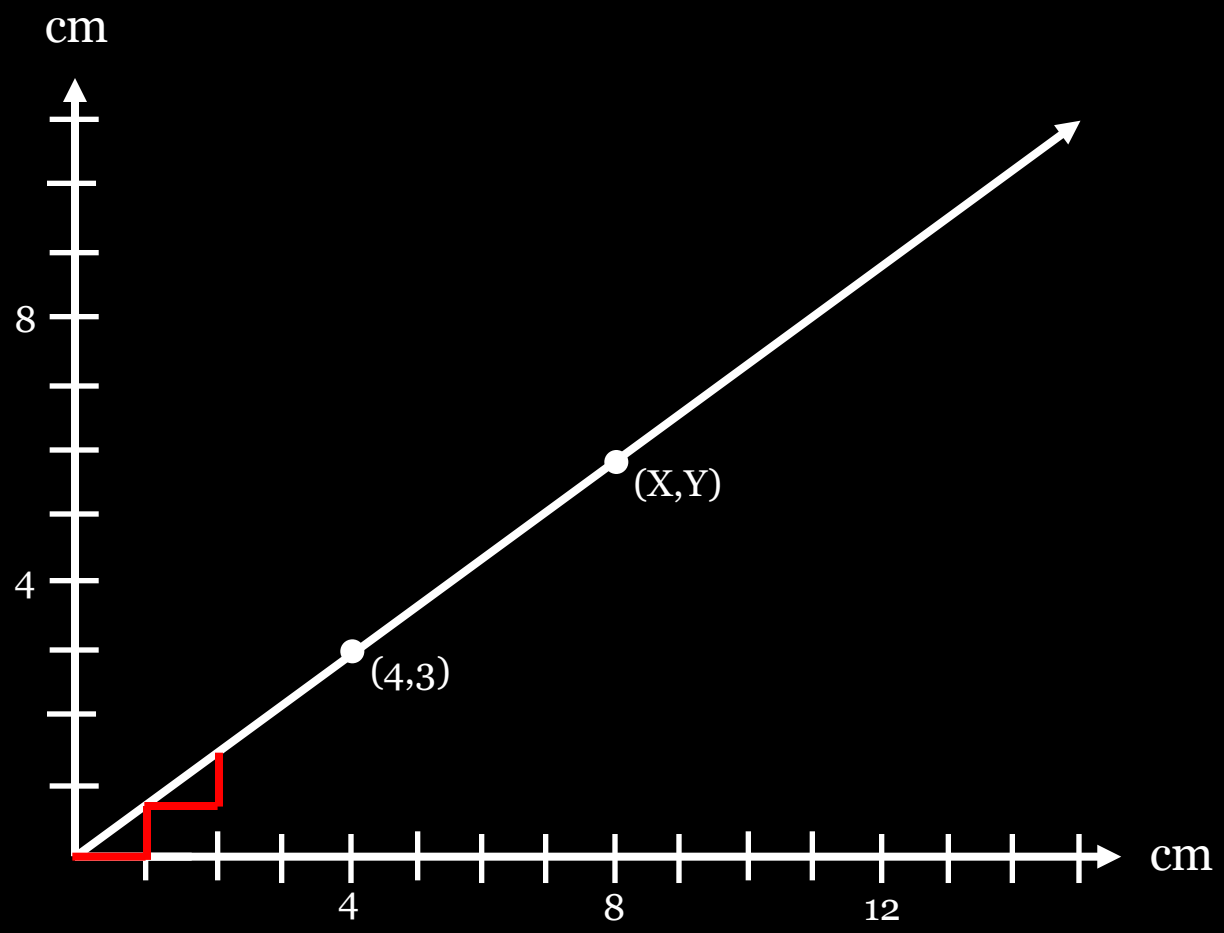
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



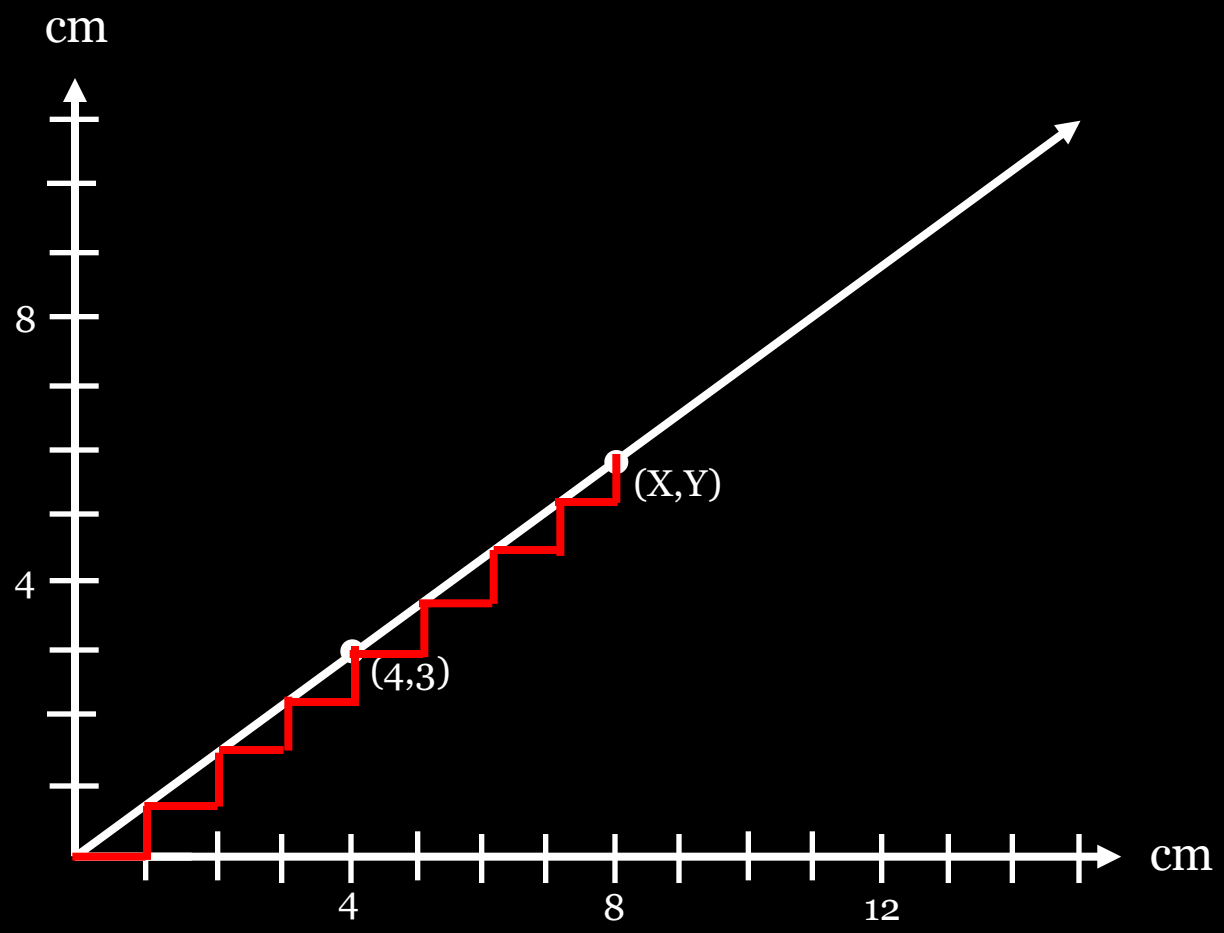
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



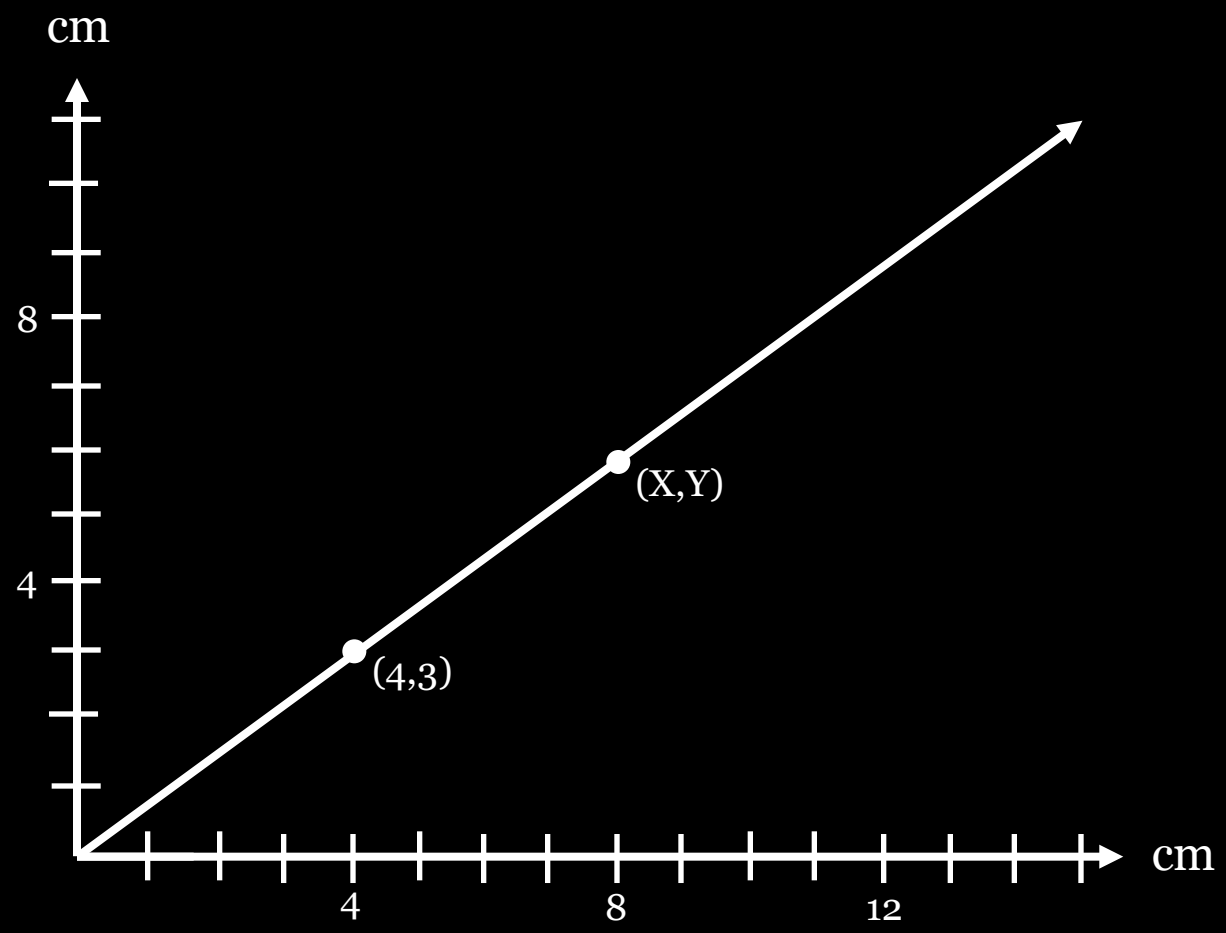
GENERATE AN EQUATION FOR A LINE

Multiply Unit-Rate Batch



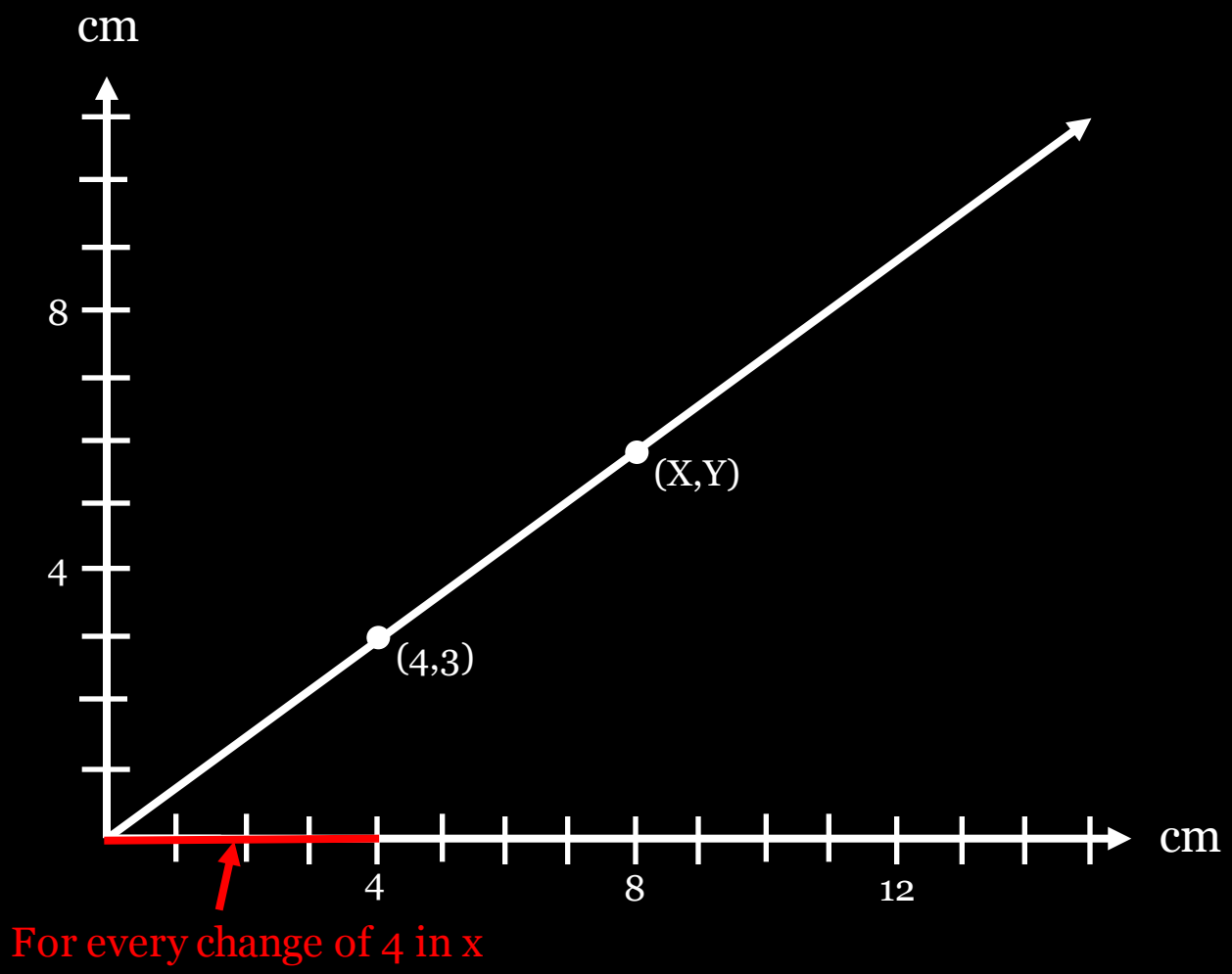
GENERATE AN EQUATION FOR A LINE

Multiply One Batch



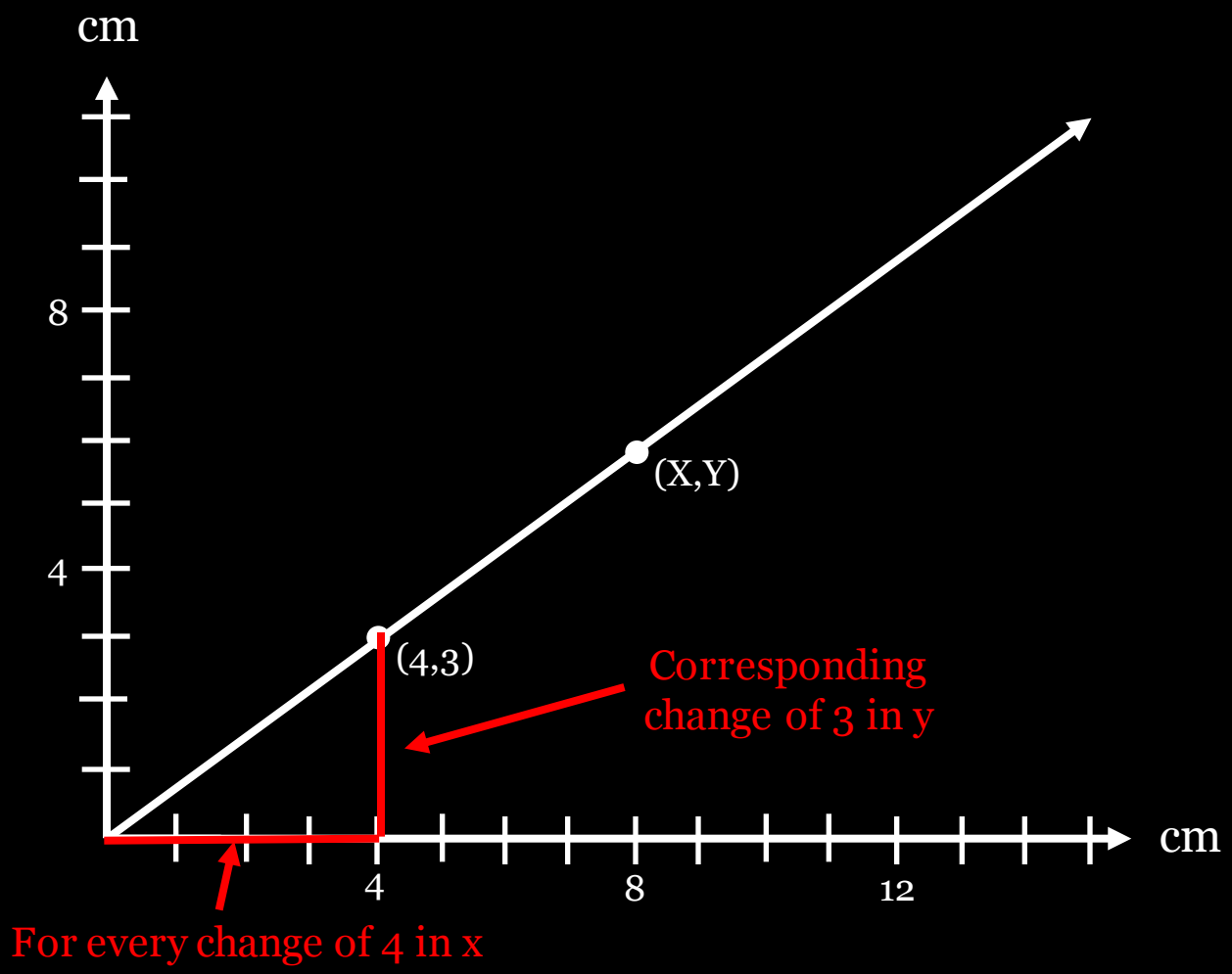
GENERATE AN EQUATION FOR A LINE

Multiply One Batch



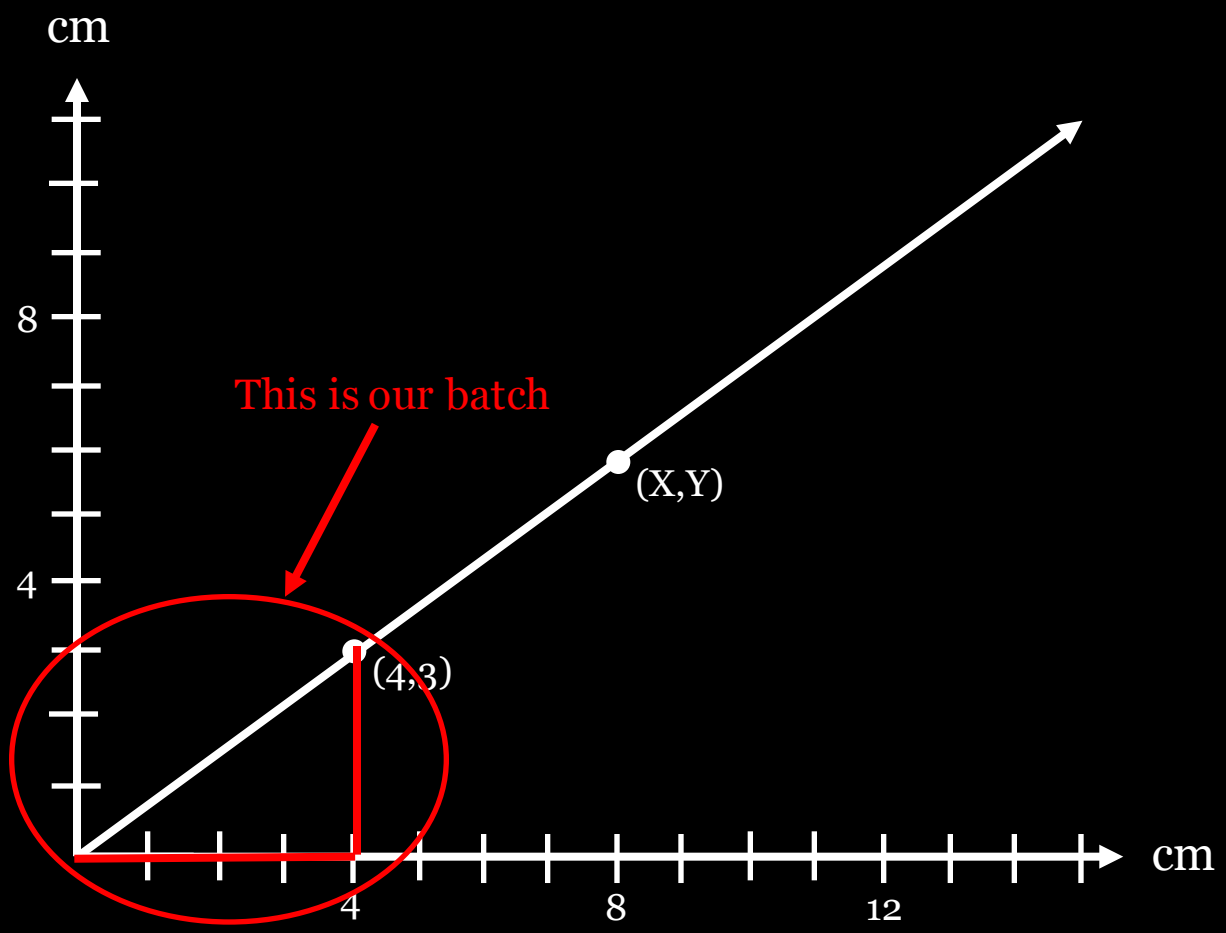
GENERATE AN EQUATION FOR A LINE

Multiply One Batch



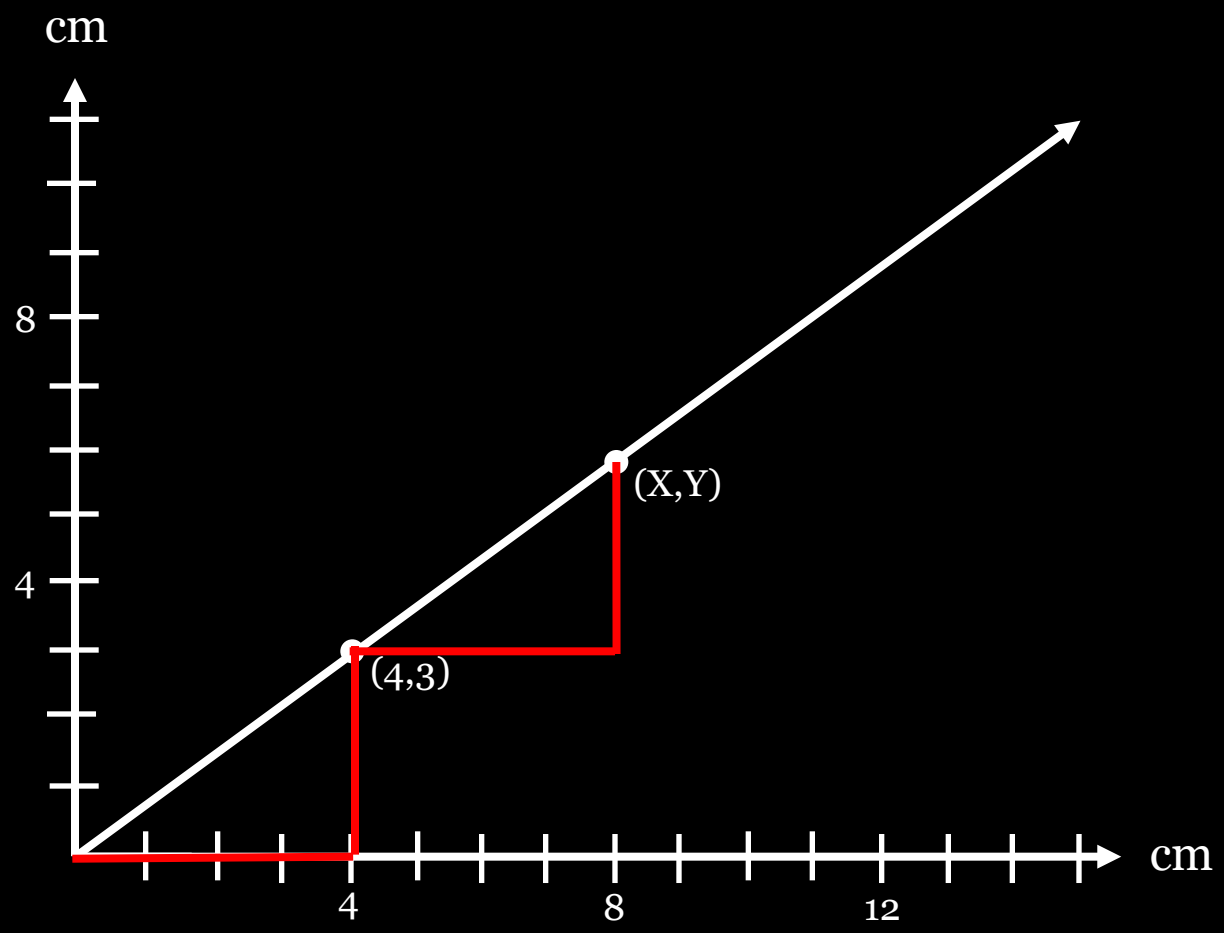
GENERATE AN EQUATION FOR A LINE

Multiply One Batch



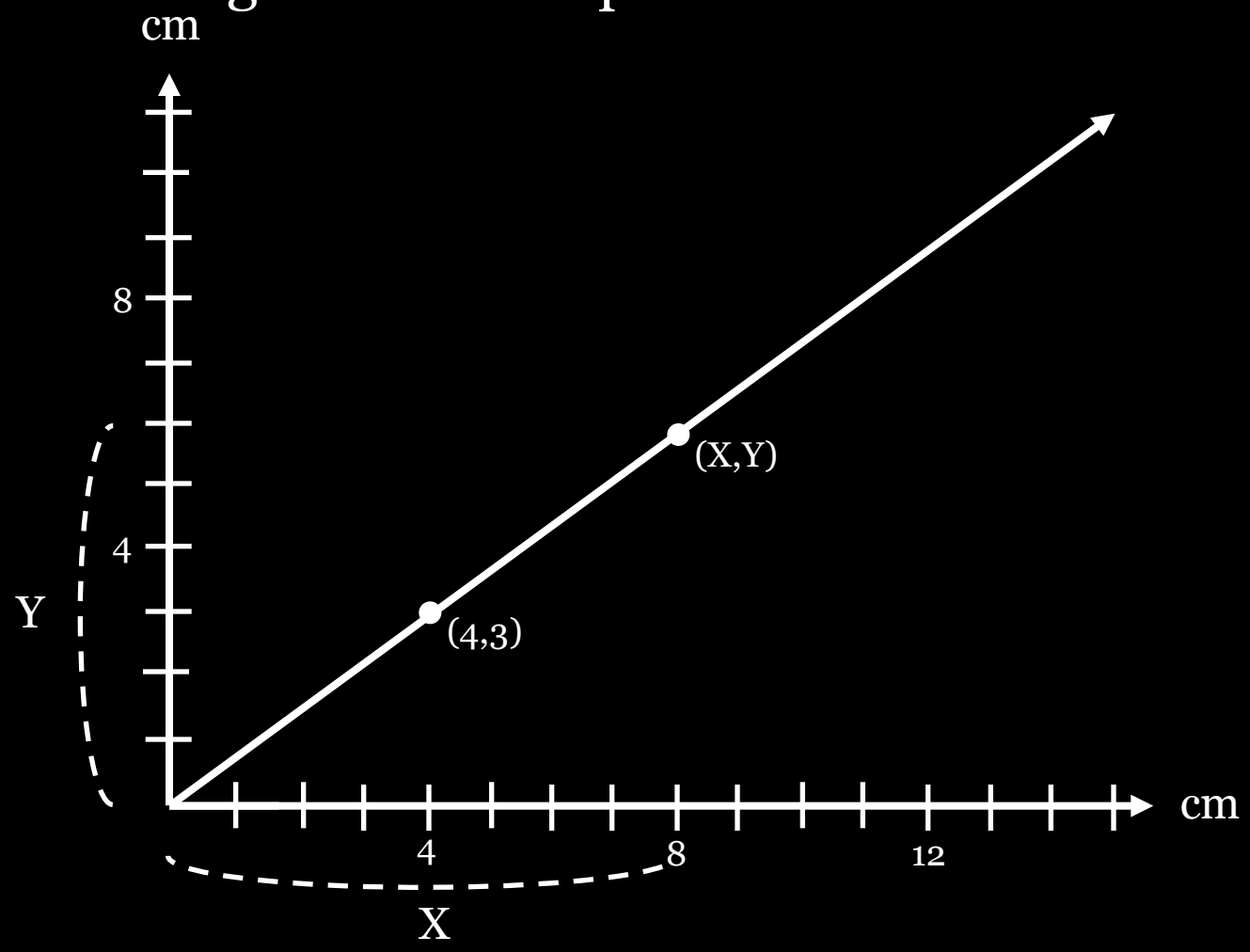
GENERATE AN EQUATION FOR A LINE

Multiply One Batch



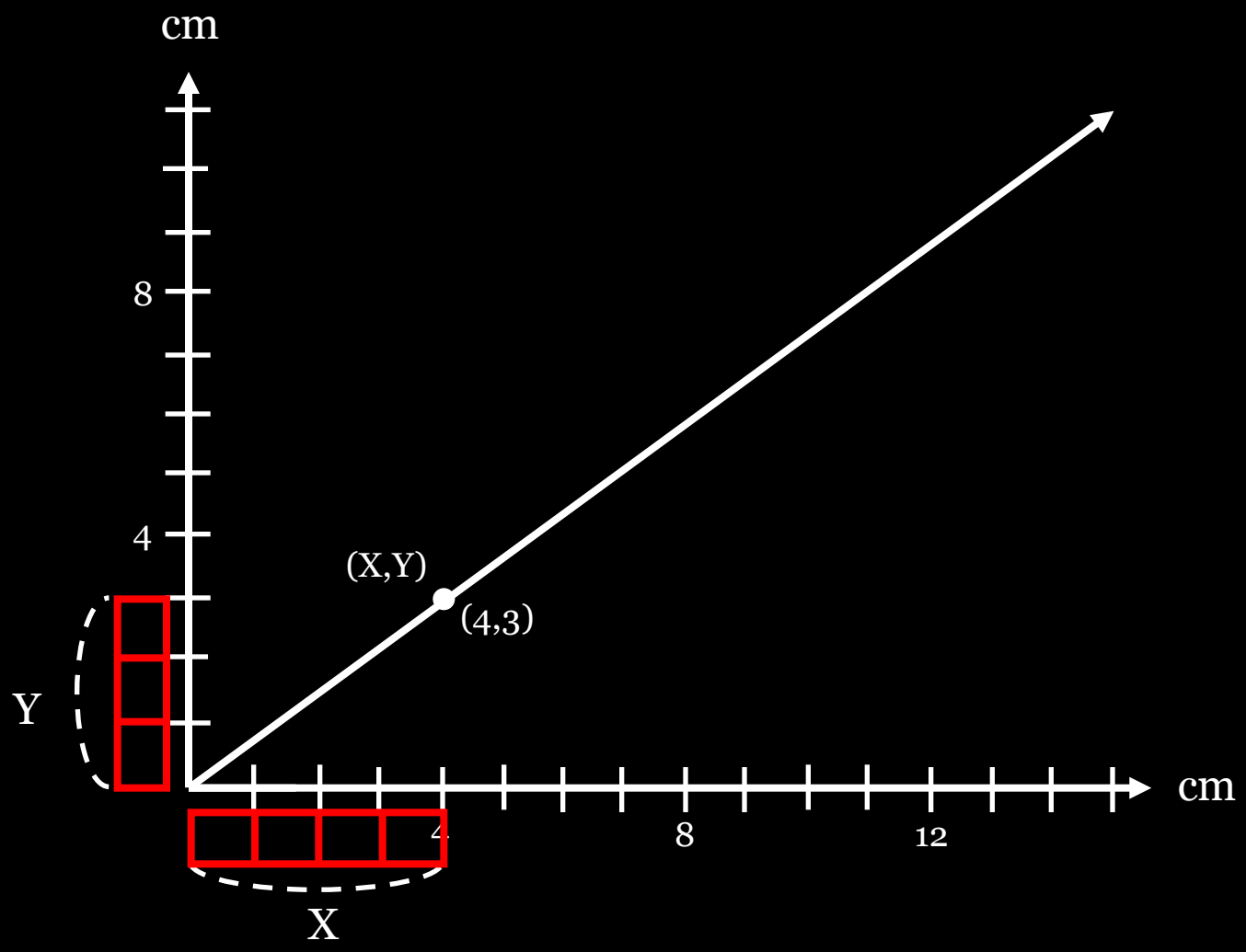
GENERATE AN EQUATION FOR A LINE

Could you use the variable parts perspective to generate an equation for the line?



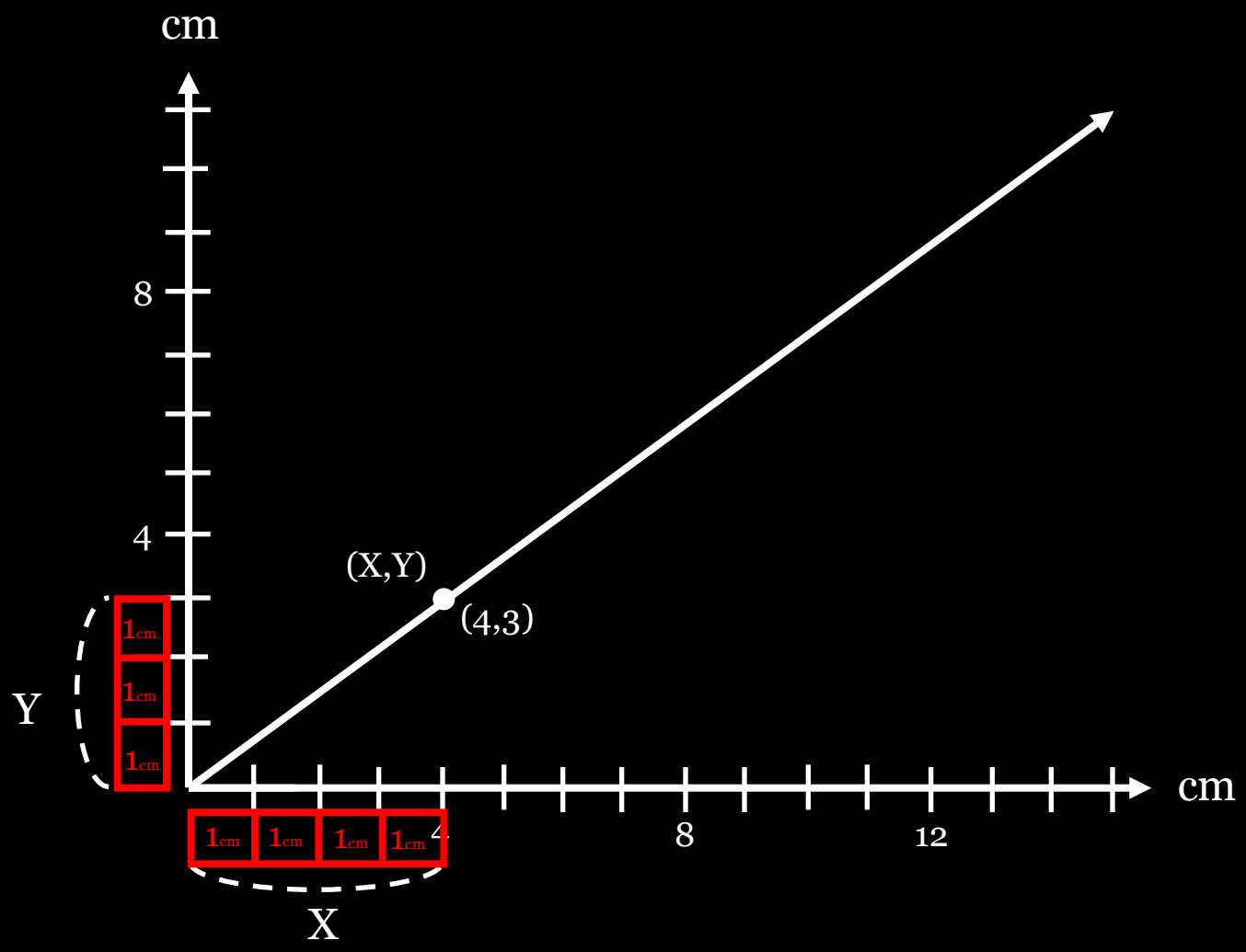
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



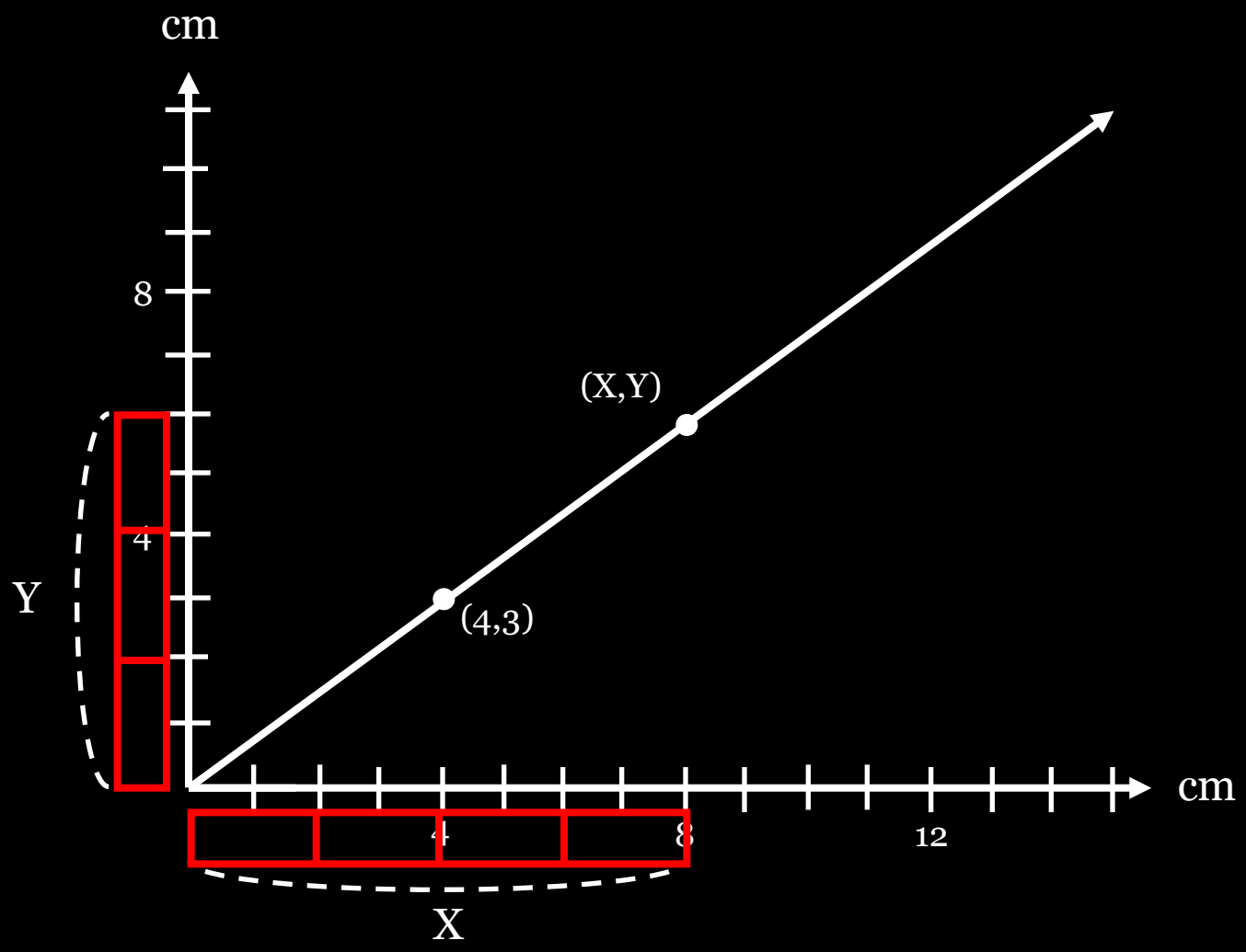
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



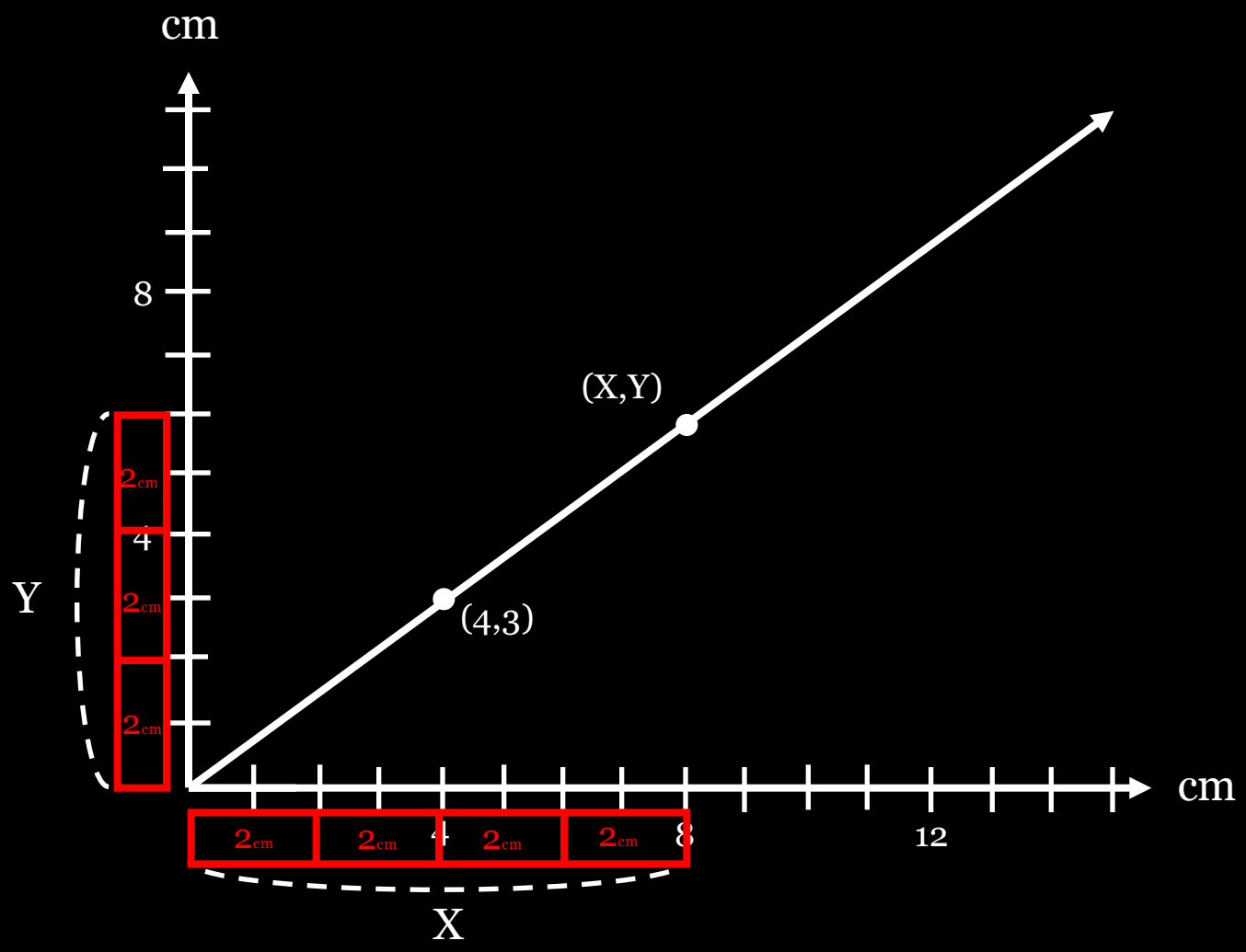
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



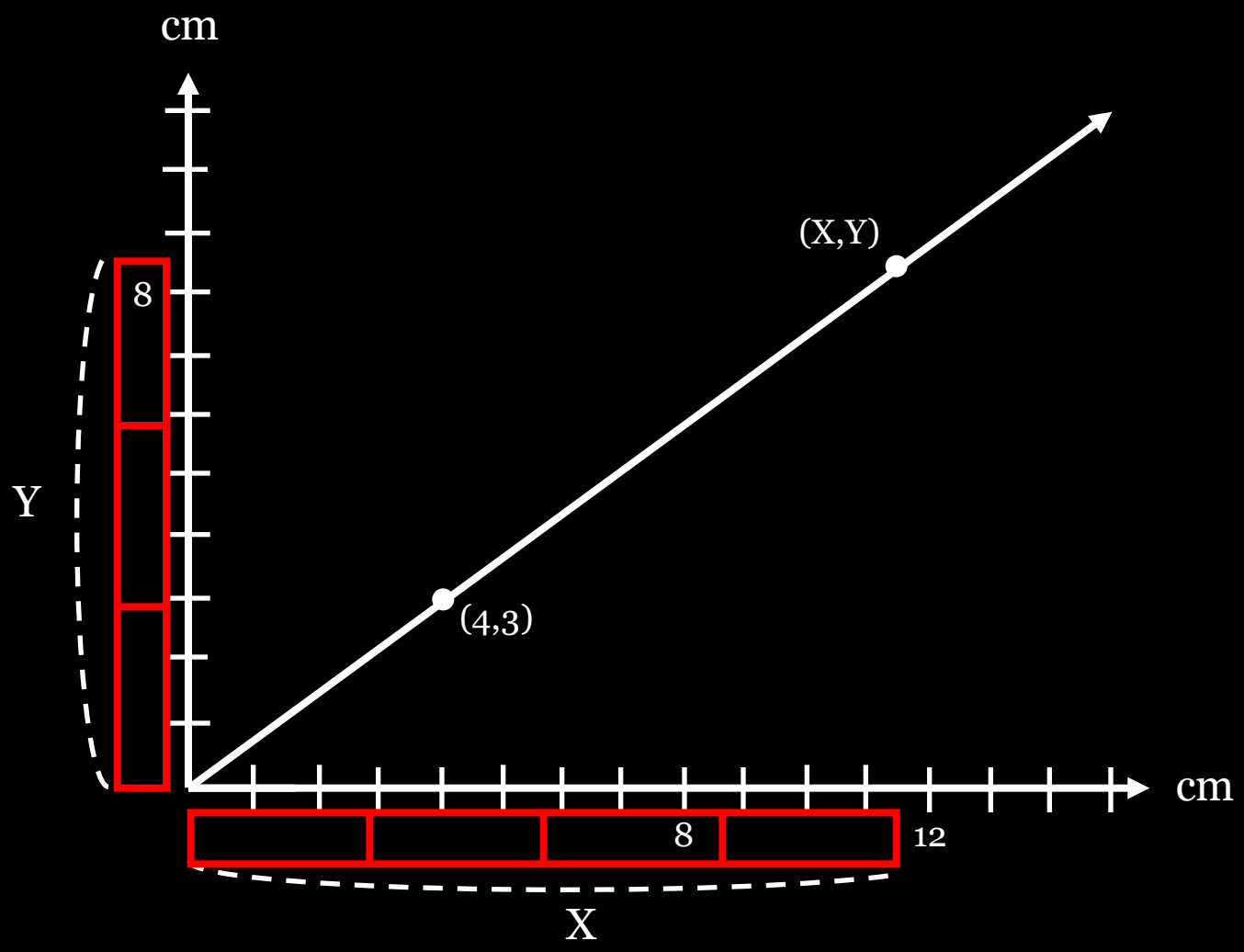
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



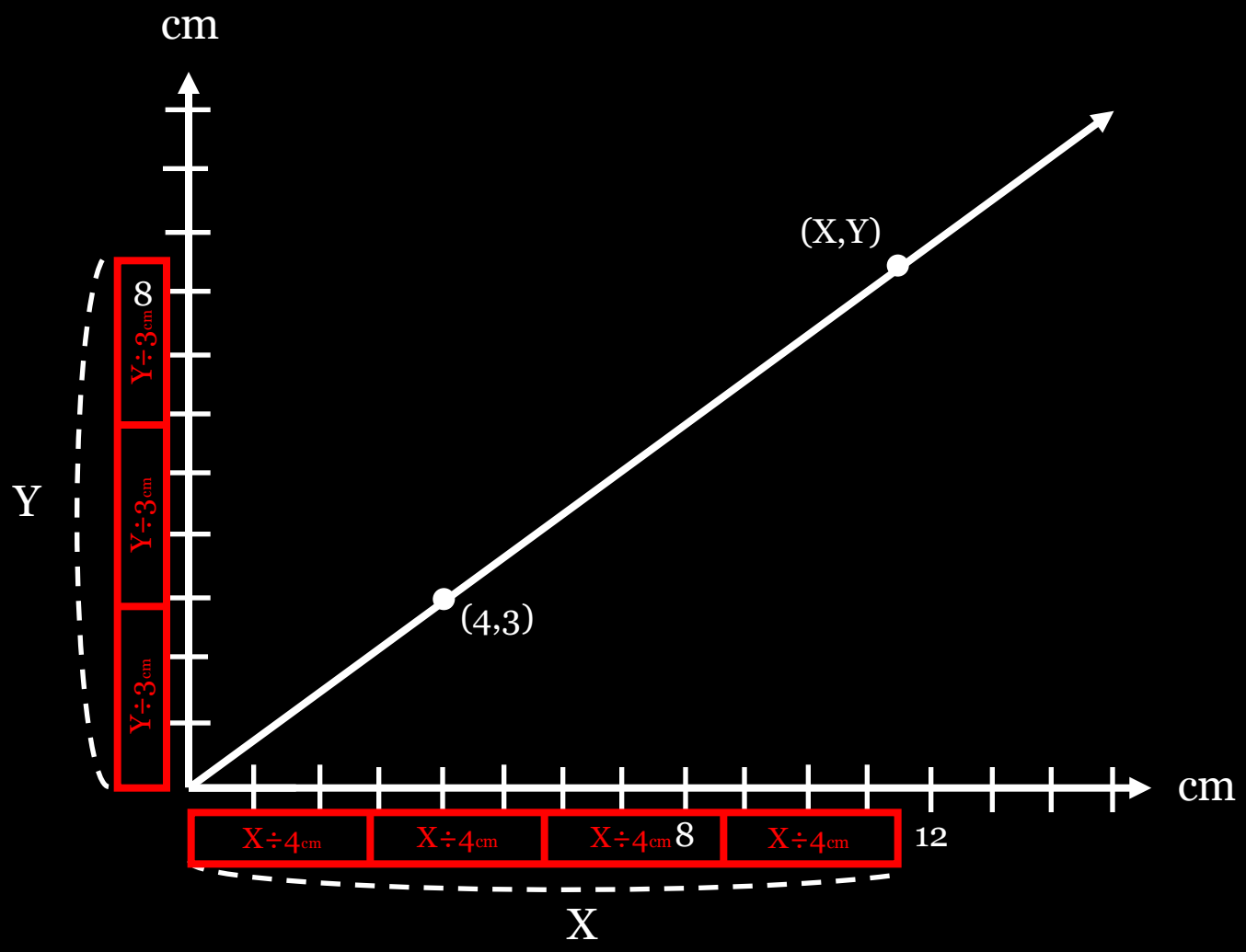
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



GENERATE AN EQUATION FOR A LINE

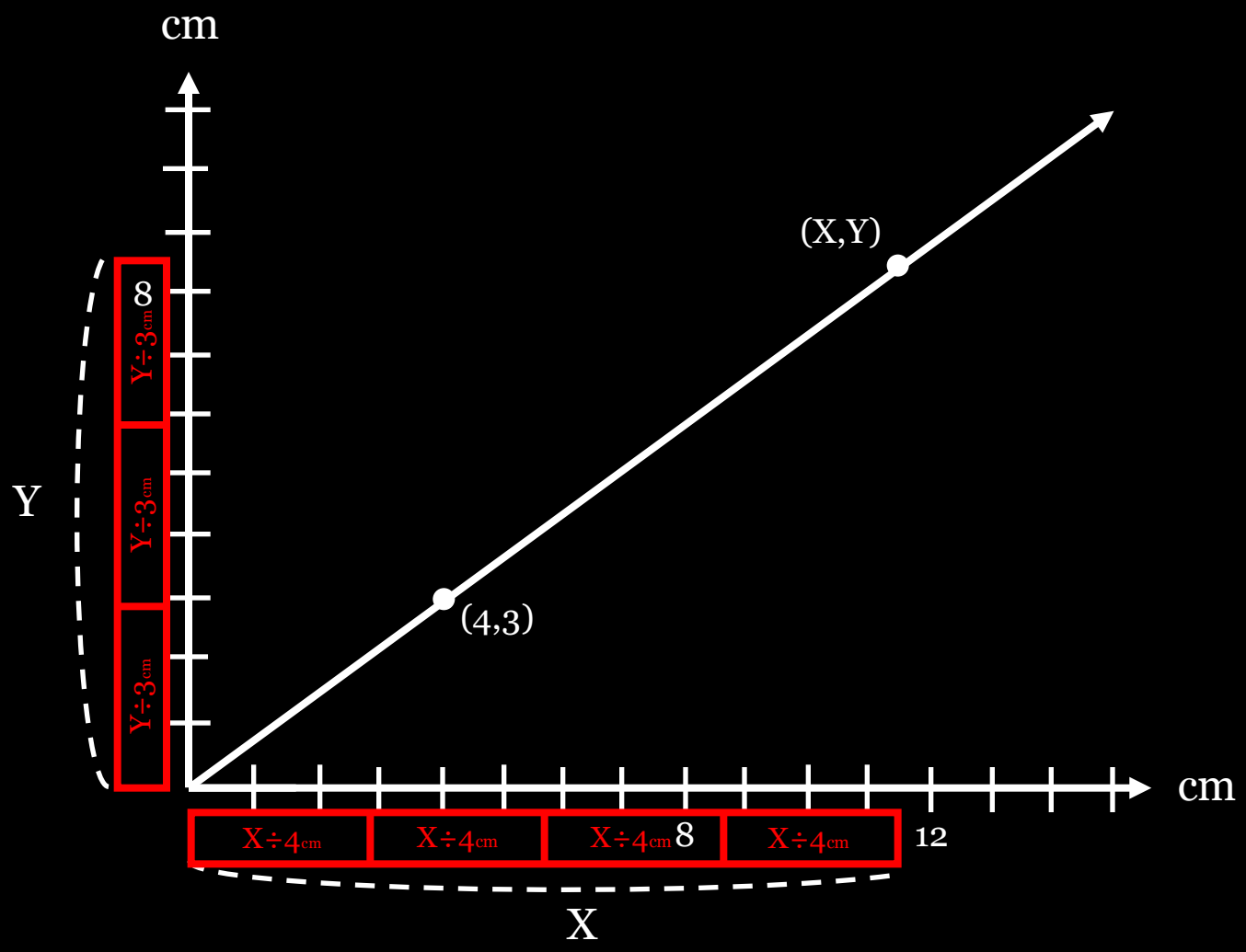
Variable Parts Perspective



We pause our presentation
to present you with a
GeoGebra sketch

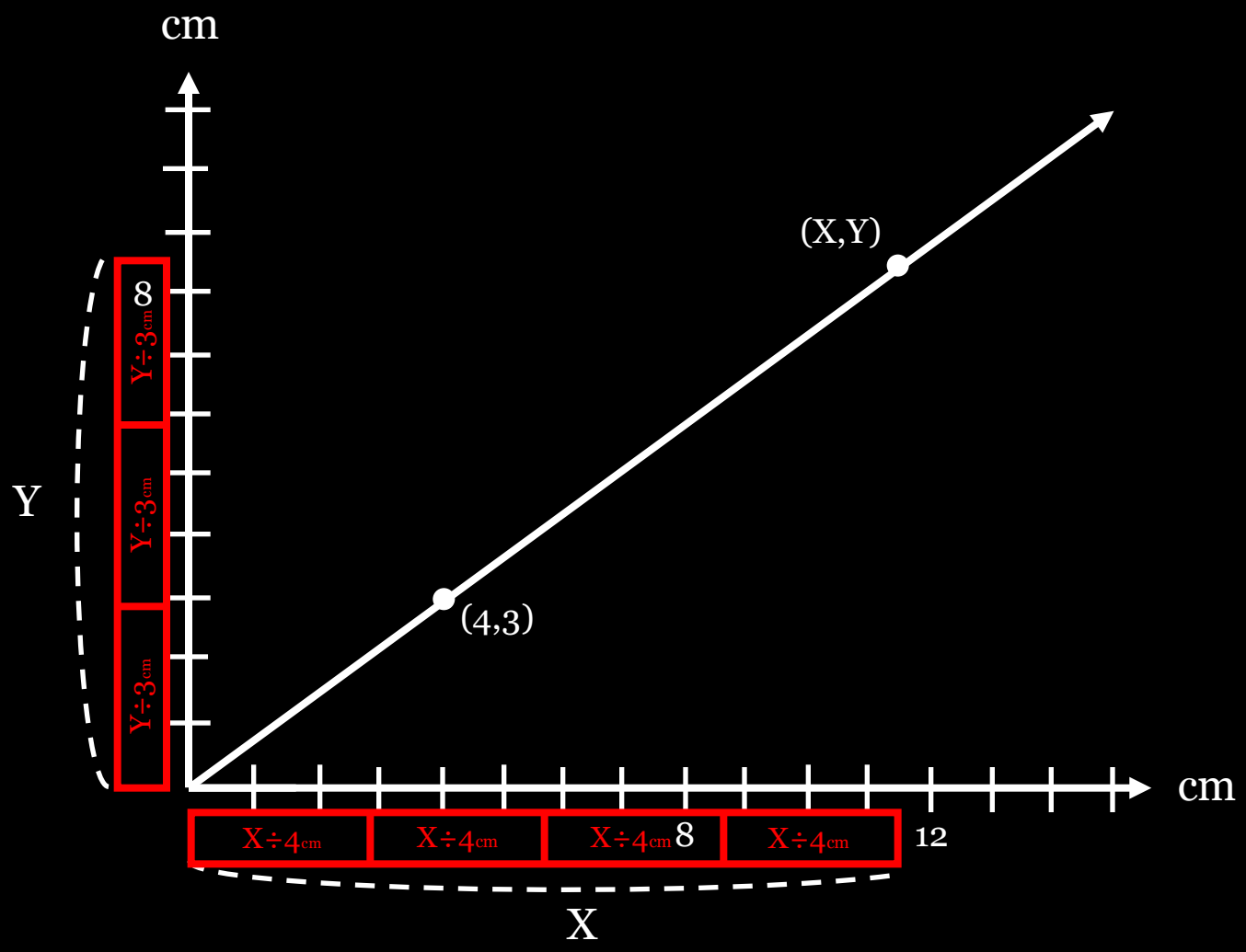
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



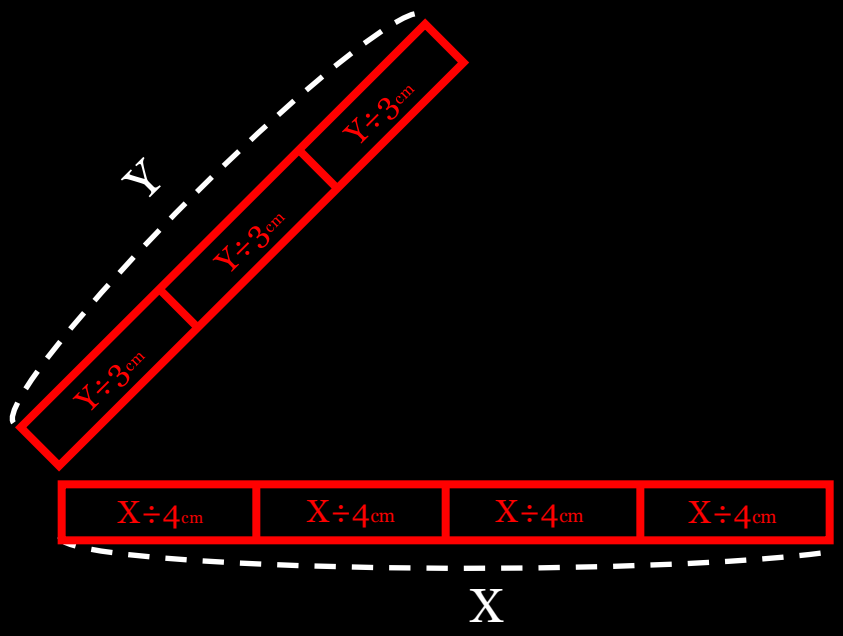
GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective



GENERATE AN EQUATION FOR A LINE

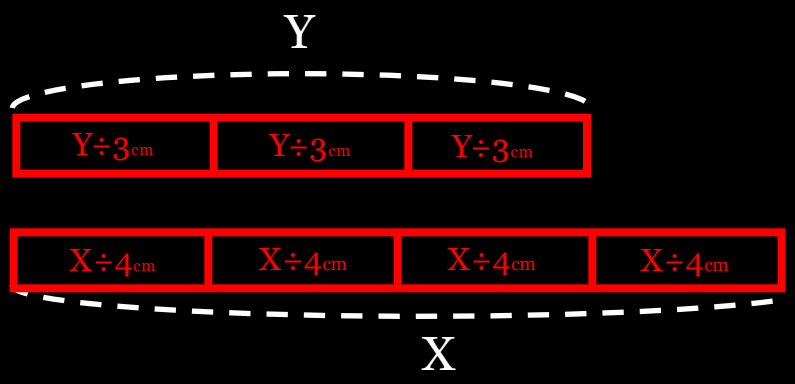
Variable Parts Perspective



GENERATE AN EQUATION FOR A LINE

Variable Parts Perspective

$$y = \frac{3}{4} \cdot x$$



DRAWING INFERENCE FROM A RANDOM SAMPLE

DRAWING INFERENCE FROM A RANDOM SAMPLE

Random Sample Problem

There is a bag of 200 tiles, some yellow, the rest blue.
You take a random sample of 10 tiles.
3 are yellow, 7 are blue.

About how many yellow tiles do you expect in the bag?

DRAWING INFERENCE FROM A RANDOM SAMPLE

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Solve the *Random Sample Problem* in two ways by reasoning
(1) from the multiple batches perspective and
(2) from the variable parts perspective.

DRAWING INFERENCE FROM A RANDOM SAMPLE

Multiple Batches Perspective

The probability of winning a game is 7%.
What does that mean? What happens if you play a lot of times?

DRAWING INFERENCE FROM A RANDOM SAMPLE

Multiple Batches Perspective

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What does that mean? What happens if you play a lot of times?

Multiple Batches Thinking

Every 100 times you play, you expect to win about 7 games.

100 games ⟶ about 7 wins

100 games ⟶ about 7 wins

100 games ⟶ about 7 wins

.
.
.

DRAWING INFERENCE FROM A RANDOM SAMPLE

Variable Parts Perspective

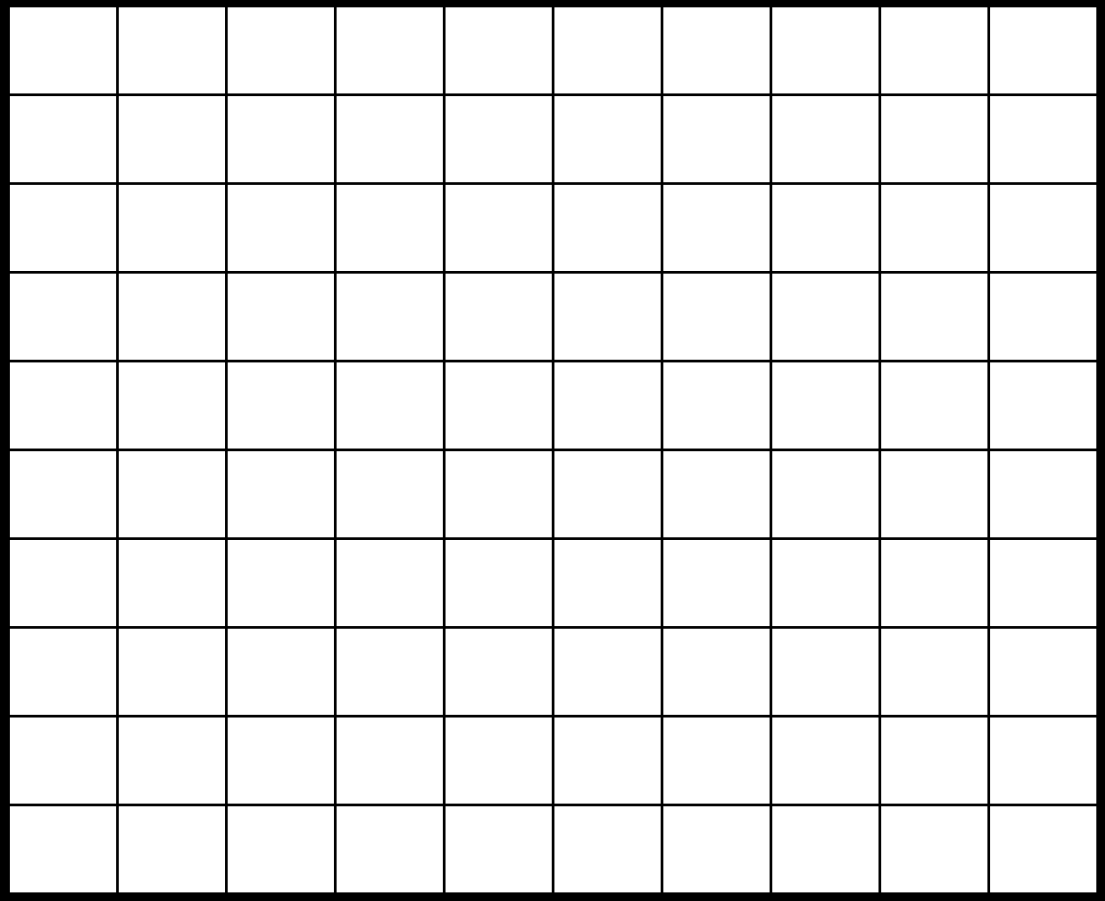
The probability of winning a game is 7%.
What does that mean? What happens if you play a lot of times?

Variable Parts Thinking

Think of 100 parts, 7 representing wins. When you play, the game falls randomly into a part.

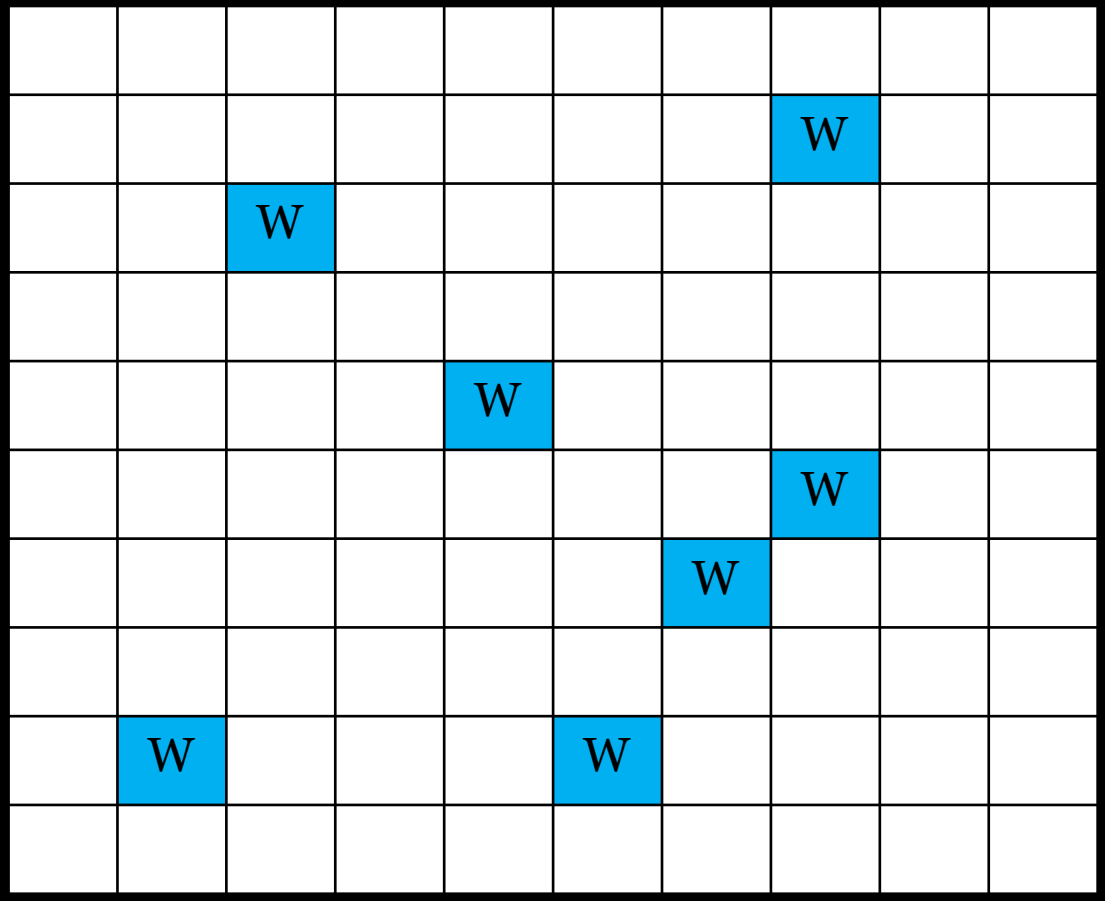
DRAWING INFERENCE FROM A RANDOM SAMPLE

Variable Parts Perspective



DRAWING INFERENCE FROM A RANDOM SAMPLE

Variable Parts Perspective



FOUNDATION FOR UNDERSTANDING TRIGONOMETRY

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Variable Parts Perspective

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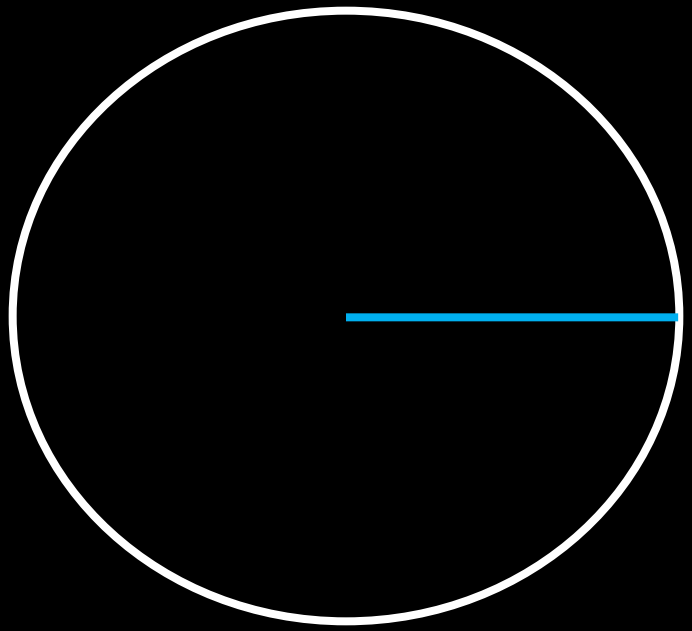
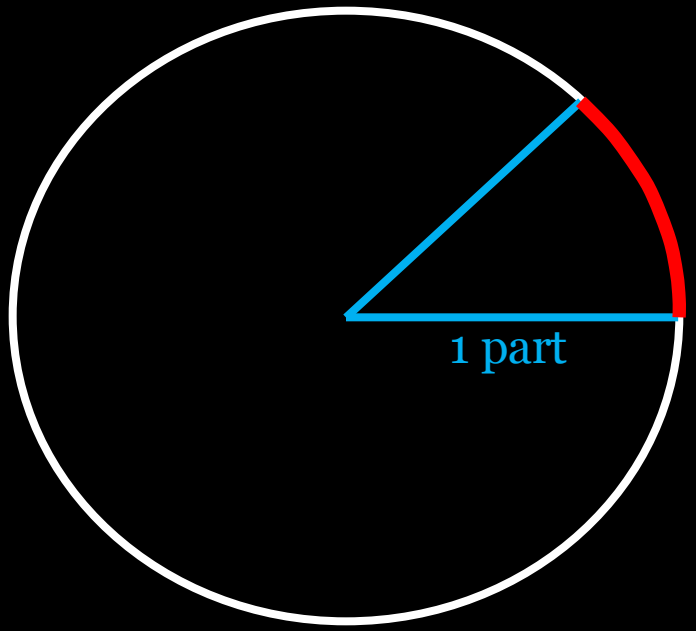
Variable Parts Perspective



Let the radius of the circle
be 1 part ...

FOUNDATION FOR UNDERSTANDING TRIGONOMETRY

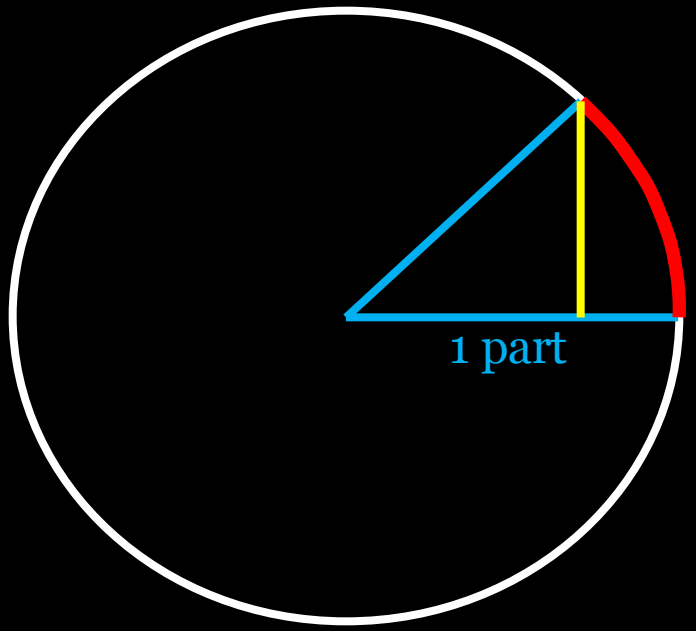
Variable Parts Perspective



… then the length of the arc
is $\pi/6$ parts …

FOUNDATION FOR UNDERSTANDING TRIGONOMETRY

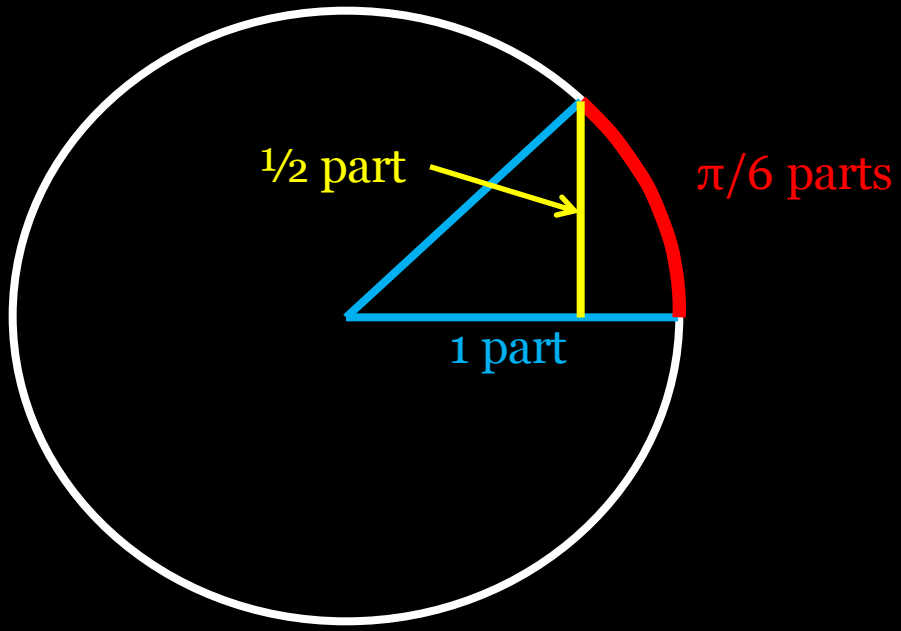
Variable Parts Perspective



… and the length of this segment must be ½ part.

FOUNDATION FOR UNDERSTANDING TRIGONOMETRY

Variable Parts Perspective



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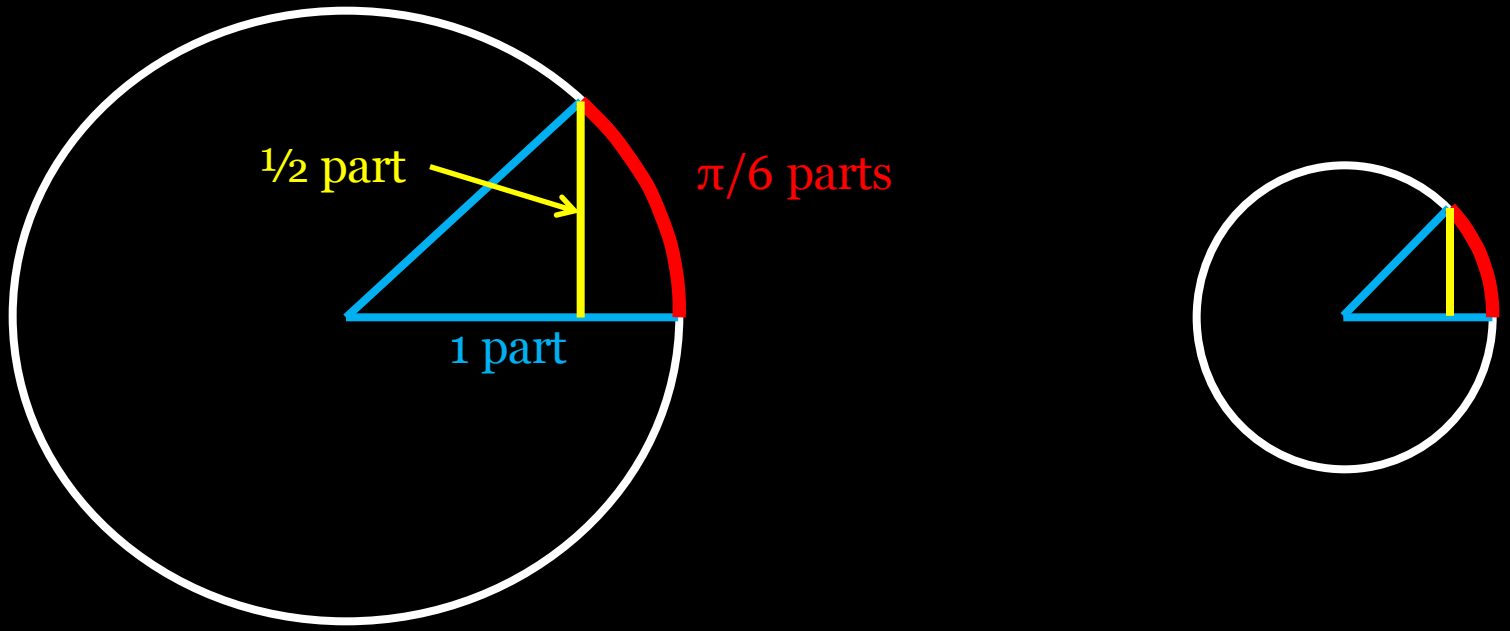
Variable Parts Perspective



Questions? Comments! Concerns😬

Come see our other sessions!

- **Friday 10:15-11:30, Salon 7**: *The Development and Negotiation of Sociomathematical Norms for Drawings*